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On Preliminary Processing of Substrate to Decrease of Mismatch-Induced Stresses in a Heterostructure

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Abstract

In this paper, we introduce an approach to reduce mismatch-induced stress in a heterostructure by using radiation processing of the substrate prior to epitaxial layer growth from the gas phase. Within the framework of the approach, mismatch-induced stresses in the heterostructure can also accelerate recombination and the diffusion of radiation defects from the damaged region during growth from the gas phase at high temperature. We also consider an analytical approach for analyzing mass and heat transfer. The approach allows simultaneous consideration of spatial and temporal variations in the parameters of mass transfer. At the same time, the approach allows for the consideration of the nonlinearity of the processes under consideration.

Keywords: Heterostructure, Mismatch-induced stresses, Radiation processing, Growth from gas phase, Radiation processing analytical approach for analyzing mass and heat transfer.

1 | Introduction

The widespread use of multilayer structures for the manufacturing of solid-state electronic devices necessitates improving the properties of their constituent epitaxial layers. At present, several technological processes are used to grow multilayer structures: molecular beam epitaxy, gas-phase epitaxy, and magnetron sputtering. A large number of works consider the growth and use of heterostructures, given their widespread use [1–10]. It is well known that heterostructures experience strain due to lattice mismatch. In this paper, we consider the possibility of reducing mismatch-induced stress by radiation-processing the substrate before epitaxial layer growth. After that, we consider the heterostructure during its growth in a reactor from the gas phase (see *Fig. 1*). We also introduce an analytical approach for analyzing mass and heat transport. The approach allows simultaneous consideration of both spatial and temporal changes in mass-transfer parameters. The approach also allows consideration of the nonlinearity of the processes.

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a.

b.

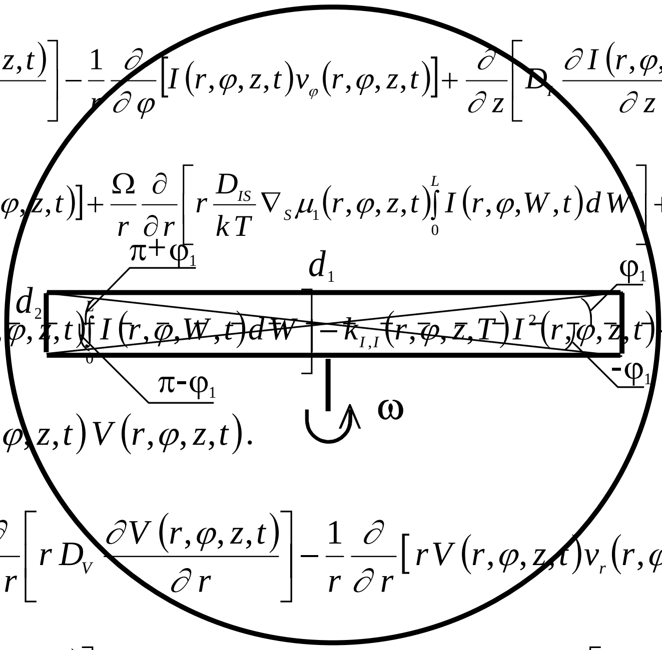


Fig. 1. Gas-phase epitaxy reactor with a sloping substrate holder: a. reactor structure, b. side view of the holder and its linear approximation with slope angle φ1.

2 | Method of Solution

To achieve our aim, we calculate the spatio-temporal distributions of radiation defect concentrations by solving the following system of equations [11–14].

$$\begin{aligned}
\frac{\partial I(r,\varphi,z,t)}{\partial t} &= \frac{1}{r}\frac{\partial}{\partial r}\left[r D_I \frac{\partial I(r,\varphi,z,t)}{\partial r}\right] - \frac{1}{r}\frac{\partial}{\partial r}\left[r I(r,\varphi,z,t) v_r(r,\varphi,z,t)\right] + \\
&+ \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[D_I \frac{\partial I(r,\varphi,z,t)}{\partial \varphi}\right] - \frac{1}{r}\frac{\partial}{\partial \varphi}\left[I(r,\varphi,z,t) v_\varphi(r,\varphi,z,t)\right] + \frac{\partial}{\partial z}\left[D_I \frac{\partial I(r,\varphi,z,t)}{\partial z}\right] - \\
&- \frac{\partial}{\partial z}\left[I(r,\varphi,z,t) v_z(r,\varphi,z,t)\right] + \frac{\Omega}{r}\frac{\partial}{\partial r}\left[r \frac{D_{IS}}{kT} \nabla_S \mu_1(r,\varphi,z,t) \int_0^L I(r,\varphi,W,t)\, dW\right] + \\
&+ \frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left[\frac{D_{IS}}{kT} \nabla_S \mu_1(r,\varphi,z,t) \int_0^L I(r,\varphi,W,t)\, dW\right] - k_{I,I}(r,\varphi,z,T) I^2(r,\varphi,z,t) - \\
&- k_{I,V}(r,\varphi,z,T) I(r,\varphi,z,t) V(r,\varphi,z,t).
\end{aligned} \quad (1)$$

$$\begin{aligned}
\frac{\partial V(r,\varphi,z,t)}{\partial t} &= \frac{1}{r}\frac{\partial}{\partial r}\left[r D_V \frac{\partial V(r,\varphi,z,t)}{\partial r}\right] - \frac{1}{r}\frac{\partial}{\partial r}\left[r V(r,\varphi,z,t) v_r(r,\varphi,z,t)\right] + \\
&+ \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[D_V \frac{\partial V(r,\varphi,z,t)}{\partial \varphi}\right] - \frac{1}{r}\frac{\partial}{\partial \varphi}\left[V(r,\varphi,z,t) v_\varphi(r,\varphi,z,t)\right] + \frac{\partial}{\partial z}\left[D_V \frac{\partial V(r,\varphi,z,t)}{\partial z}\right] - \\
&- \frac{\partial}{\partial z}\left[V(r,\varphi,z,t) v_z(r,\varphi,z,t)\right] + \frac{\Omega}{r}\frac{\partial}{\partial r}\left[r \frac{D_{VS}}{kT} \nabla_S \mu_1(r,\varphi,z,t) \int_0^L V(r,\varphi,W,t)\, dW\right] + \\
&+ \frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left[\frac{D_{VS}}{kT} \nabla_S \mu_1(r,\varphi,z,t) \int_0^L V(r,\varphi,W,t)\, dW\right] - k_{V,V}(r,\varphi,z,T) V^2(r,\varphi,z,t) - \\
&- k_{I,V}(r,\varphi,z,T) I(r,\varphi,z,t) V(r,\varphi,z,t).
\end{aligned}$$

with boundary and initial conditions

$$I(r,\varphi,z,0)=f_I(r,\varphi,z),\ I(0,\varphi,z,t)\neq\infty,\ \left.\frac{\partial I(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0,\ \left.\frac{\partial I(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0,$$

$$\left.\frac{\partial I(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0,\ V(r,\varphi,z,0)=f_I(r,\varphi,z),\ V(0,\varphi,z,t)\neq\infty,$$

$$\left.\frac{\partial V(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0, \tag{2}$$

$$\left.\frac{\partial V(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0,\ I(r,-\varphi_1,z,t)=I(r,\varphi_1,z,t)=I(r,\pi-\varphi_1,z,t)=$$

$$=I(r,\pi+\varphi_1,z,t),\ V(r,-\varphi_1,z,t)=V(r,\varphi_1,z,t)=V(r,\pi-\varphi_1,z,t)=V(r,\pi+\varphi_1,z,t).$$

Here I (r,φ,z,t) is the spatio-temporal distribution of concentration of radiation interstitials with equilibrium distribution V^*; V (r,φ,z,t) is the spatio-temporal distribution of concentration of radiation vacancies with equilibrium distribution V^*; $D_I(r,\varphi,z, T)$, $D_V(r,\varphi,z, T)$, $D_{IS}(r,\varphi,z, T)$, $D_{VS}(r,\varphi,z, T)$ are the coefficients of volumetric and surficial diffusions of interstitials and vacancies, respectively; terms $V^2(r,\varphi,z,t)$ and $I^2(r,\varphi,z,t)$ correspond to generation of divacancies and diinterstitials, respectively (see, for example, [14] and appropriate references in this book); $k_{I,V}(r,\varphi,z, T)$, $k_{I,I}(r,\varphi,z, T)$ and $k_{V,V}(r,\varphi,z, T)$ are the parameters of recombination of point radiation defects and generation of their complexes. Spatio-temporal distributions of divacancies Φ_V (r, z, t)) and diinterstitials Φ_I (r, z, t)) could be determined by solving the following system of equations [13], [14].

$$\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial r}\right]+k_I(r,\varphi,z,T)I(r,\varphi,z,t)+$$

$$+k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t)+\frac{1}{r}\frac{\partial}{\partial \varphi}\left[D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial \varphi}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial z}\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L\Phi_I(r,\varphi,W,t)dW\right]+ \tag{3}$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L\Phi_I(r,\varphi,W,t)dW\right].$$

$$\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial r}\right]+k_V(r,\varphi,z,T)V(r,\varphi,z,t)+$$

$$+k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)+\frac{1}{r}\frac{\partial}{\partial \varphi}\left[D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial \varphi}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial z}\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L \Phi_V(r,\varphi,W,t)dW\right]+$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L \Phi_V(r,\varphi,W,t)dW\right].$$

with boundary and initial conditions

$$\left.\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0,\ \left.\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0,$$

$$\left.\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0,\ \left.\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0, \tag{4}$$

$\Phi_V(r,\varphi,z,0)=0$, $\Phi_V(0,\varphi,z,t)\neq\infty$, $\Phi_I(r,\varphi,z,0)=0$, $\Phi_I(0,\varphi,z,t)\neq\infty$,

$\Phi_I(r,-\varphi_1,z,t)=\Phi_I(r,\varphi_1,z,t)=\Phi_I(r,\pi-\varphi_1,z,t)=\Phi_I(r,\pi+\varphi_1,z,t)$, $\Phi_V(r,-\varphi_1,z,t)=$

$=\Phi_V(r,\varphi_1,z,t)=\Phi_V(r,\pi-\varphi_1,z,t)=\Phi_V(r,\pi+\varphi_1,z,t)$.

Here $D_{\Phi I}(x,y,z,T)$, $D_{\Phi V}(x,y,z,T)$, $D_{\Phi IS}(x,y,z,T)$ and $D_{\Phi VS}(x,y,z,T)$ are the coefficients of volumetric and surficial diffusions of complexes of radiation defects; $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ are the parameters of decay of complexes of radiation defects.

Chemical potential μ_1 in *Eqs. (1)* and *(2)* can be determined by the following relation [11].

$$\mu_1=E(z)\Omega\sigma_{ij}\,[u_{ij}(r,\varphi,z,t)+u_{ji}(r,\varphi,z,t)]/2, \tag{5}$$

where $E(z)$ is the Young modulus, σ_{ij} is the stress tensor; $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ is the deformation tensor; u_i, u_j are the components $u_r(r,\varphi,z,t)$, $u_\varphi(r,\varphi,z,t)$ and $u_z(r,\varphi,z,t)$ of the displacement vector $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$; x_i, x_j are the coordinate r,φ,z. *Eq. (5)* could be transformed to the following form

$$\mu_1(r,\varphi,z,t)=E(z)\frac{\Omega}{2}\left[\frac{\partial u_i(r,\varphi,z,t)}{\partial x_j}+\frac{\partial u_j(r,\varphi,z,t)}{\partial x_i}\right]\left\{\frac{1}{2}\left[\frac{\partial u_i(r,\varphi,z,t)}{\partial x_j}+\frac{\partial u_j(r,\varphi,z,t)}{\partial x_i}\right]-\right.$$

$$\left.-\varepsilon_0\delta_{ij}+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\left[\frac{\partial u_k(r,\varphi,z,t)}{\partial x_k}-3\varepsilon_0\right]-K(z)\beta(z)[T(r,\varphi,z,t)-T_0]\,\delta_{ij}\right\},$$

where σ is the Poisson coefficient; $\varepsilon_0 = (a_s - a_{EL})/a_{EL}$ is the mismatch parameter; a_s, a_{EL} are lattice distances of the substrate and the epitaxial layer; K is the modulus of uniform compression; β is the coefficient of thermal expansion; T_r is the equilibrium temperature, which coincides (for our case) with room temperature. Components of the displacement vector can be obtained by solving the following equations [12].

$$\rho(z)\frac{\partial^2 u_x(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\frac{\partial\left[r\cdot\sigma_{rr}(r,\varphi,z,t)\right]}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\varphi}(r,\varphi,z,t)}{\partial\varphi}+\frac{\partial\sigma_{rz}(r,\varphi,z,t)}{\partial z},$$

$$\rho(z)\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\frac{\partial\left[r\cdot\sigma_{\varphi r}(r,\varphi,z,t)\right]}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\varphi\varphi}(r,\varphi,z,t)}{\partial y}+\frac{\partial\sigma_{\varphi z}(r,\varphi,z,t)}{\partial z},$$

$$\rho(z)\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\frac{\partial\left[r\cdot\sigma_{zx}(r,\varphi,z,t)\right]}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{zy}(r,\varphi,z,t)}{\partial\varphi}+\frac{\partial\sigma_{zz}(r,\varphi,z,t)}{\partial z},$$

where $\sigma_{ij}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial u_i(r,\varphi,z,t)}{\partial x_j}+\frac{\partial u_j(r,\varphi,z,t)}{\partial x_i}-\frac{\delta_{ij}}{3}\frac{\partial u_k(r,\varphi,z,t)}{\partial x_k}\right]+K(z)\,\delta_{ij}\times$ $\times\frac{\partial u_k(r,\varphi,z,t)}{\partial x_k}-\beta(z)K(z)\left[T(r,\varphi,z,t)-T_r\right]$, (z) is the density of materials of heterostructure, δ_{ij} is the Kronecker symbol. Taking into account the relation for σ_{ij} last system of equations could be written as

$$\rho(z)\frac{\partial^2 u_r(r,\varphi,z,t)}{\partial t^2}=\frac{1}{r}\left\{K(z)+\frac{5E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial u_r(r,\varphi,z,t)}{\partial x}\right]+\frac{\partial^2\left[r\cdot u_\varphi(r,\varphi,z,t)\right]}{\partial r\,\partial\varphi}\times$$

$$\times\frac{1}{r^2}\left\{K(z)-\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}+\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{1}{r^2}\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial z^2}\right]+$$

$$+\frac{1}{r}\left\{K(z)+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial r\,\partial z}-K(z)\beta(z)\frac{1}{r}\frac{\partial\left[r\cdot T(r,\varphi,z,t)\right]}{\partial r},$$

$$\rho(z)\frac{\partial^2 u_y(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial r}\right]+\frac{\partial}{\partial r}\left[r\frac{\partial u_r(r,\varphi,z,t)}{\partial r}\right]\right\}- \tag{6}$$

$$-K(z)\frac{\beta(z)}{r}\frac{\partial\left[r\cdot T(r,\varphi,z,t)\right]}{\partial\varphi}+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial z}+\frac{1}{r}\frac{\partial u_z(r,\varphi,z,t)}{\partial\varphi}\right]\right\}+$$

$$+\frac{1}{r^2}\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+K(z)\right\}\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial\varphi^2}+\frac{1}{r}\left\{K(z)-\frac{E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_y(r,\varphi,z,t)}{\partial\varphi\,\partial z}+$$

$$+\frac{K(z)}{r}\frac{\partial^2\left[r\cdot u_\varphi(r,\varphi,z,t)\right]}{\partial r\,\partial\varphi}.$$

$$\rho(z)\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_z(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u_z(r,\varphi,z,t)}{\partial \varphi^2}+\right.$$

$$\left.+\frac{\partial^2[r\cdot u_r(r,\varphi,z,t)]}{\partial r\partial z}+\frac{1}{r}\frac{\partial^2 u_\varphi(r,\varphi,z,t)}{\partial\varphi\partial z}\right\}+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{1}{r}\frac{\partial u_r(r,\varphi,z,t)}{\partial r}+\frac{1}{r}\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial\varphi}+\right.\right.$$

$$\left.\left.+\frac{\partial u_r(r,\varphi,z,t)}{\partial z}\right]\right\}+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(r,\varphi,z,t)}{\partial z}-\frac{1}{r}\frac{\partial[r\cdot u_x(r,\varphi,z,t)]}{\partial x}-\right.\right.$$

$$\left.\left.-\frac{1}{r}\frac{\partial u_\varphi(r,\varphi,z,t)}{\partial y}-\frac{\partial u_z(r,\varphi,z,t)}{\partial z}\right]\right\}-K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial z}.$$

The conditions for the system of the above equations can be written in the form.

$$\left.\frac{\partial\,\vec{u}(r,\varphi,z,t)}{\partial r}\right|_{r=0}=0;\ \left.\frac{\partial\,\vec{u}(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0;\ \left.\frac{\partial\,\vec{u}(r,\varphi,z,t)}{\partial z}\right|_{z=0}=0;$$

$$\left.\frac{\partial\,\vec{u}(r,\varphi,z,t)}{\partial z}\right|_{z=L}=0;$$

$$\vec{u}(r,0,z,t)=\vec{u}(r,2\pi,z,t);\ \vec{u}(r,\varphi,z,0)=\vec{u}_0;\ \vec{u}(r,\varphi,z,\infty)=\vec{u}_0.$$

We determine the spatio-temporal distributions of radiation defect concentrations by solving *Eqs. (1)* and *(2)* in the framework of the standard method of averaging of function corrections [15–18]. Previously, we transformed *Eqs. (1)* and *(2)* to the following form, taking into account initial distributions of the considered concentrations:

$$\frac{\partial I(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\,D_I\frac{\partial I(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_I\frac{\partial I(r,\varphi,z,t)}{\partial\varphi}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_I\frac{\partial I(r,\varphi,z,t)}{\partial z}\right]-\frac{1}{r}\frac{\partial}{\partial r}\left[r\,I(r,\varphi,z,t)v_r(r,\varphi,z,t)\right]-$$

$$-\frac{1}{r}\frac{\partial}{\partial\varphi}\left[I(r,\varphi,z,t)v_\varphi(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L I(r,\varphi,W,t)\,dW\right]- \tag{1.a}$$

$$-\frac{\partial}{\partial z}\left[I(r,\varphi,z,t)v_z(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L I(r,\varphi,W,t)\,dW\right]-$$

$$-k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t)-k_{I,V}(r,\varphi,z,T)I(r,\varphi,z,t)V(r,\varphi,z,t)+f_I(r,\varphi,z)\delta(t),$$

$$\frac{\partial V(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\,D_V\frac{\partial V(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[D_V\frac{\partial V(r,\varphi,z,t)}{\partial \varphi}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_V\frac{\partial V(r,\varphi,z,t)}{\partial z}\right]-\frac{1}{r}\frac{\partial}{\partial r}\left[r\,V(r,\varphi,z,t)v_r(r,\varphi,z,t)\right]-$$

$$-\frac{1}{r}\frac{\partial}{\partial \varphi}\left[V(r,\varphi,z,t)v_\varphi(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L V(r,\varphi,W,t)\,dW\right]-$$

$$-\frac{\partial}{\partial z}\left[V(r,\varphi,z,t)v_z(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L V(r,\varphi,W,t)\,dW\right]-$$

$$-k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)-k_{I,V}(r,\varphi,z,T)I(r,\varphi,z,t)V(r,\varphi,z,t)+f_V(r,\varphi,z)\delta(t) \qquad \textbf{(2.a)}$$

$$\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial r}\right]+k_I(r,\varphi,z,T)\,I(r,\varphi,z,t)+$$

$$+\frac{1}{r}\frac{\partial}{\partial \varphi}\left[D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial \varphi}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_I(r,\varphi,z,t)}{\partial z}\right]+$$

$$+k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t)+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L \Phi_I(r,\varphi,W,t)\,dW\right]+$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left[\frac{D_{\Phi IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L \Phi_I(r,\varphi,W,t)\,dW\right],$$

$$\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial r}\right]+k_V(r,\varphi,z,T)\,V(r,\varphi,z,t)+$$

$$+\frac{1}{r}\frac{\partial}{\partial \varphi}\left[D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial \varphi}\right]+\frac{\partial}{\partial z}\left[D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_V(r,\varphi,z,t)}{\partial z}\right]+$$

$$+k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)+\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L \Phi_V(r,\varphi,W,t)\,dW\right]+$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\int_0^L \Phi_V(r,\varphi,W,t)\,dW\right].$$

Further, we replace the radiation defect concentrations on the right-hand sides of *Eqs. (1.a)* and *(2.a)* with their not yet known average values $\alpha_{1\rho}$. In this situation, we obtain equations for the first-order approximations of the required concentrations in the following form.

$$\frac{\partial I_1(r,\varphi,z,t)}{\partial t}=\alpha_{1I}L\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]+\alpha_{1I}L\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]-$$

$$-\frac{\alpha_{1I}}{r}\frac{\partial\left[r\cdot v_r(r,\varphi,z,t)\right]}{\partial r}-\frac{\alpha_{1I}}{r}\frac{\partial v_\varphi(r,\varphi,z,t)}{\partial\varphi}-\alpha_{1I}\frac{\partial v_z(r,\varphi,z,t)}{\partial z}-\alpha_{1I}^2k_{I,I}(r,\varphi,z,T)- \quad (1.b)$$

$$-\alpha_{1I}\alpha_{1V}k_{I,V}(r,\varphi,z,T)+f_I(r,\varphi,z)\,\delta(t),$$

$$\frac{\partial V_1(r,\varphi,z,t)}{\partial t}=\alpha_{1V}L\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]+\alpha_{1V}L\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]-$$

$$-\frac{\alpha_{1V}}{r}\frac{\partial\left[r\cdot v_r(r,\varphi,z,t)\right]}{\partial r}-\frac{\alpha_{1V}}{r}\frac{\partial v_\varphi(r,\varphi,z,t)}{\partial\varphi}-\alpha_{1V}\frac{\partial v_z(r,\varphi,z,t)}{\partial z}-\alpha_{1V}^2k_{V,V}(r,\varphi,z,T)-$$

$$-\alpha_{1I}\alpha_{1V}k_{I,V}(r,\varphi,z,T)+f_V(r,\varphi,z)\,\delta(t), \quad (2.b)$$

$$\frac{\partial\Phi_{1I}(r,\varphi,z,t)}{\partial t}=\alpha_{1\Phi_I}L\frac{\Omega}{r}\left\{\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[r\frac{D_{\Phi_IS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]\right\}+$$

$$+k_I(r,\varphi,z,T)I(r,\varphi,z,t)+k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t).$$

$$\frac{\partial\Phi_{1V}(r,\varphi,z,t)}{\partial t}=\alpha_{1\Phi_V}L\frac{\Omega}{r}\left\{\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[r\frac{D_{\Phi_VS}}{kT}\nabla_S\mu_1(r,\varphi,z,t)\right]\right\}+$$

$$+k_V(r,\varphi,z,T)V(r,\varphi,z,t)+k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t).$$

Integration of the left and right sides of *Eqs. (1.b)* and *(2.b)* over time gives us the possibility to obtain relations for the above approximation in the final form

$$I_1(r,\varphi,z,t)=\alpha_{1I}L\frac{\Omega}{r}\left\{\frac{\partial}{\partial r}\left[r\frac{D_{IS}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]+\frac{\partial}{\partial\varphi}\left[\frac{D_{IS}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]\right\}-$$

$$-\frac{\alpha_{1I}}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t v_r(r,\varphi,z,\tau)d\tau\right]-\frac{\alpha_{1I}}{r}\frac{\partial}{\partial\varphi}\int_0^t v_\varphi(r,\varphi,z,\tau)d\tau-\alpha_{1I}\frac{\partial}{\partial z}\int_0^t v_z(r,\varphi,z,\tau)d\tau- \quad (1.b)$$

$$-\alpha_{1I}^2\int_0^t k_{I,I}(r,\varphi,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(r,\varphi,z,T)d\tau+f_I(r,\varphi,z),$$

$$V_1(r,\varphi,z,t)=\alpha_{1V}L\frac{\Omega}{r}\left\{\frac{\partial}{\partial r}\left[r\frac{D_{VS}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]+\frac{\partial}{\partial\varphi}\left[\frac{D_{VS}}{kT}\nabla_S\int_0^t\mu_1(r,\varphi,z,\tau)d\tau\right]\right\}- \quad (2.b)$$

$$-\frac{\alpha_{1V}}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t v_r(r,\varphi,z,\tau)d\tau\right]-\frac{\alpha_{1V}}{r}\frac{\partial}{\partial\varphi}\int_0^t v_\varphi(r,\varphi,z,\tau)d\tau-\alpha_{1V}\frac{\partial}{\partial z}\int_0^t v_z(r,\varphi,z,\tau)d\tau-$$

$$-\alpha_{1V}^2\int_0^t k_{V,V}(r,\varphi,z,T)d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(r,\varphi,z,T)d\tau+f_V(r,\varphi,z),$$

$$\Phi_{1I}(r,\varphi,z,t)=\alpha_{1\Phi_I}L\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\int_0^t \mu_1(r,\varphi,z,\tau)d\tau\right]+\int_0^t k_I(r,\varphi,z,T)I(r,\varphi,z,\tau)d\tau+$$

$$+\alpha_{1\Phi_I}L\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[r\frac{D_{\Phi_I S}}{kT}\nabla_S\int_0^t \mu_1(r,\varphi,z,\tau)d\tau\right]+\int_0^t k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,\tau)d\tau,$$

$$\Phi_{1V}(r,\varphi,z,t)=\alpha_{1\Phi_V}L\frac{\Omega}{r}\frac{\partial}{\partial r}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\int_0^t \mu_1(r,\varphi,z,\tau)d\tau\right]+\int_0^t k_V(r,\varphi,z,T)V(r,\varphi,z,\tau)d\tau+$$

$$+\alpha_{1\Phi_V}L\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left[r\frac{D_{\Phi_V S}}{kT}\nabla_S\int_0^t \mu_1(r,\varphi,z,\tau)d\tau\right]+\int_0^t k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,\tau)d\tau\cdot$$

We determine the average values of the first-order approximations to the concentrations of radiation defects using the following *Relation (16)* [15–18].

$$\alpha_1=\frac{1}{\pi\Theta R^2L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\rho_1\,dz\,d\varphi\,dr\,dt. \tag{7}$$

Substitution of the *Relations (1b)* and *(2b)* into *Relation (7)* gives us the possibility to obtain the required average values in the following form.

$$\alpha_{1I}=\sqrt{\frac{(a_3+A)^2}{4a_4^2}-4\left(B+\frac{\Theta a_3B+2\pi LR^2\Theta^2a_1}{a_4}\right)}-\frac{a_3+A}{4a_4},$$

$$\alpha_{1V}=\frac{1}{S_{IV00}}\left[\frac{\Theta}{\alpha_{1I}}\int_0^{2\pi}\int_0^L\int_0^R r\cdot f_\rho(r,\varphi,z)dr\,dz\,d\varphi-\alpha_{1I}S_{II00}-2\pi LR^2\Theta\right],$$

where $S_{\rho\rho ij}=\int_0^\Theta(\Theta-t)\int_0^{2\pi}\int_0^L\int_0^R k_{\rho,\rho}(r,\varphi,z,T)I_1^i(r,\varphi,z,t)V_1^j(r,\varphi,z,t)dr\,dz\,d\varphi\,dt$, $a_4=S_{II00}\times$ $\times\left(S_{IV00}^2-S_{II00}S_{VV00}\right)$, $a_3=S_{IV00}S_{II00}+S_{IV00}^2-S_{II00}S_{VV00}$, $a_2=\int_0^{2\pi}\int_0^L\int_0^R r\cdot f_\rho(r,\varphi,z)dr\,dz\,d\varphi\times$ $\times S_{IV00}S_{IV00}^2+4\pi^2L^2R^2\Theta S_{IV00}+2S_{II00}S_{VV00}\int_0^{2\pi}\int_0^L\int_0^R r\cdot f_\rho(r,\varphi,z)dr\,dz\,d\varphi-4\pi^2L^2R^2\Theta S_{VV00}-$ $-S_{IV00}^2\int_0^{2\pi}\int_0^L\int_0^R r\cdot f_\rho(r,\varphi,z)dr\,dz\,d\varphi$, $a_1=S_{IV00}\int_0^{2\pi}\int_0^L\int_0^R r\cdot f_\rho(r,\varphi,z)dr\,dz\,d\varphi$, $a_0=S_{VV00}\times$ $\times\left[\int_0^{2\pi}\int_0^L\int_0^R r\cdot f_\rho(r,\varphi,z)dr\,dz\,d\varphi\right]^2$, $A=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}}$, $B=\frac{\Theta a_2}{6a_4}+\sqrt[3]{\sqrt{q^2+p^3}-q}-$

$$-\sqrt[3]{\sqrt{q^2+p^3}+q}\,,\ q=\frac{\Theta^3 a_2}{24a_4^2}\left(4a_0-2\pi LR\Theta\frac{a_1a_3}{a_4}\right)-\Theta^2\frac{a_0}{8a_4^2}\left(4\Theta a_2-\Theta^2\frac{a_3^2}{a_4}\right)-$$

$$-\Theta^3\left(a_2^3+27\pi^2L^2R^2\Theta a_1^2a_4\right)/54a_4^3,\ p=\left[3a_4\Theta^2\left(2a_0a_4-\pi LR\Theta a_1a_3\right)-\Theta a_2a_4\right]/18a_4,$$

$$\alpha_{1\Phi_I}=\frac{R_{I1}}{2\pi LR\Theta}+\frac{S_{II20}}{2\pi LR\Theta}+\frac{1}{2\pi LR}\int_0^{2\pi}\int_0^L\int_0^R r\cdot f_\rho(r,\varphi,z)\,dr\,dz\,d\varphi$$

$$\alpha_{1\Phi_V}=\frac{R_{V1}}{2\pi LR\Theta}+\frac{S_{VV20}}{2\pi LR\Theta}+\frac{1}{2\pi LR}\int_0^{2\pi}\int_0^L\int_0^R r\cdot f_\rho(r,\varphi,z)\,dr\,dz\,d\varphi,$$

where $R_{\rho i}=\int_0^\Theta(\Theta-t)\int_0^{2\pi}\int_0^L\int_0^R k_I(r,\varphi,z,T)I_1^i(r,\varphi,z,t)\,dr\,dz\,d\varphi$.

We determine approximations of the second and higher orders of the concentrations of radiation defects using the standard iterative procedure of the method of averaging of function corrections [15–18]. In the framework of the procedure to determine approximations of the n-th order of concentrations of radiation defects I (r,φ,z,t), V (r,φ, z,t), Φ_I (r,φ,z,t) and $_V$ (r,φ,z,t) we replace the required concentrations in *Eqs. (1.b)* and *(2.b)* with the following sum: $_{(n\rho}+\rho_{n-1}$(r,φ,z,t). The replacement leads to the following transformation of the appropriate equations

$$\frac{\partial I_2(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_I(r,\varphi,z,T)\frac{\partial I_1(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_I(r,\varphi,z,T)\frac{\partial I_1(r,\varphi,z,t)}{\partial\varphi}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_I(r,\varphi,z,T)\frac{\partial I_1(r,\varphi,z,t)}{\partial z}\right]-k_{I,V}(r,\varphi,z,T)\left[\alpha_{1I}+I_1(r,\varphi,z,t)\right]\left[\alpha_{1V}+V_1(r,\varphi,z,t)\right]+$$
$$+\frac{\Omega}{r}\frac{\partial}{\partial r}\left\{r\cdot\frac{D_{IS}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^L\left[\alpha_{2I}+I_1(r,\varphi,W,t)\right]dW\right\}-k_{I,I}(r,\varphi,z,T)\left[\alpha_{1I}+I_1(r,\varphi,z,t)\right]^2+ \quad \textbf{(1.c)}$$
$$+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left\{\frac{D_{IS}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^L\left[\alpha_{2I}+I_1(r,\varphi,W,t)\right]dW\right\},$$

$$\frac{\partial V_2(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_V(r,\varphi,z,T)\frac{\partial V_1(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[D_V(r,\varphi,z,T)\frac{\partial V_1(r,\varphi,z,t)}{\partial\varphi}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_V(r,\varphi,z,T)\frac{\partial V_1(r,\varphi,z,t)}{\partial z}\right]-k_{I,V}(r,\varphi,z,T)\left[\alpha_{1I}+I_1(r,\varphi,z,t)\right]\left[\alpha_{1V}+V_1(r,\varphi,z,t)\right]+$$
$$+\frac{\Omega}{r}\frac{\partial}{\partial r}\left\{r\cdot\frac{D_{VS}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^L\left[\alpha_{2I}+V_1(r,\varphi,W,t)\right]dW\right\}-k_{V,V}(r,\varphi,z,T)\left[\alpha_{1V}+V_1(r,\varphi,z,t)\right]^2+ \quad \textbf{(2.c)}$$
$$+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left\{\frac{D_{VS}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^L\left[\alpha_{2V}+V_1(r,\varphi,W,t)\right]dW\right\},$$

$$\frac{\partial \Phi_{2I}(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_{1I}(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[D_{\Phi_I}(r,\varphi,z,T)\times\right.$$

$$\left.\times\frac{\partial \Phi_{1I}(r,\varphi,z,t)}{\partial \varphi}\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^L\left[\alpha_{2\Phi_I}+\Phi_{1I}(r,\varphi,W,t)\right]dW\right\}+$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(r,\varphi,W,t)\right]dW\right\}+k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,t)+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(r,\varphi,z,T)\frac{\partial \Phi_{1I}(r,\varphi,z,t)}{\partial z}\right]+k_I(r,\varphi,z,T)I(r,\varphi,z,t)+f_{\Phi_I}(r,\varphi,z)\delta(t),$$

$$\frac{\partial \Phi_{2V}(r,\varphi,z,t)}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_{1V}(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[D_{\Phi_V}(r,\varphi,z,T)\times\right.$$

$$\left.\times\frac{\partial \Phi_{1V}(r,\varphi,z,t)}{\partial \varphi}\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^L\left[\alpha_{2\Phi_V}+\Phi_{1V}(r,\varphi,W,t)\right]dW\right\}+$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(r,\varphi,z,t)\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(r,\varphi,W,t)\right]dW\right\}+k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,t)+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(r,\varphi,z,T)\frac{\partial \Phi_{1V}(r,\varphi,z,t)}{\partial z}\right]+k_V(r,\varphi,z,T)V(r,\varphi,z,t)+f_{\Phi_V}(r,\varphi,z)\delta(t)\cdot$$

Integration of the left- and right-hand sides of *Eqs. (1.c)* and *(2.c)* gives us the possibility to obtain relations for the required concentrations in the final form

$$I_2(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t D_I(r,\varphi,z,T)\frac{\partial I_1(r,\varphi,z,\tau)}{\partial r}d\tau\right]+\frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[\int_0^t D_I(r,\varphi,z,T)\times\right.$$

$$\left.\times\frac{\partial I_1(r,\varphi,z,\tau)}{\partial \varphi}d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_I(r,\varphi,z,T)\frac{\partial I_1(r,\varphi,z,\tau)}{\partial z}d\tau\right]-\int_0^t\left[\alpha_{1I}+I_1(r,\varphi,z,\tau)\right]^2\times$$

$$\times k_{I,I}(r,\varphi,z,T)\,d\tau+\frac{\Omega}{r}\frac{\partial}{\partial r}\left\{r\cdot\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2I}+I_1(r,\varphi,W,\tau)\right]dW\,d\tau\right\}+\qquad\textbf{(1.d)}$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial \varphi}\left\{\int_0^t\frac{D_{IS}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2I}+I_1(r,\varphi,W,\tau)\right]dW\,d\tau\right\}-\int_0^t k_{I,V}(r,\varphi,z,T)\times$$

$$\times\left[\alpha_{1V}+I_1(r,\varphi,z,\tau)\right]\left[\alpha_{1V}+V_1(r,\varphi,z,\tau)\right]d\tau\,,$$

$$V_2(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t D_V(r,\varphi,z,T)\frac{\partial V_1(r,\varphi,z,\tau)}{\partial r}d\tau\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[\int_0^t D_V(r,\varphi,z,T)\times\right.$$

$$\left.\times\frac{\partial V_1(r,\varphi,z,\tau)}{\partial\varphi}d\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_V(r,\varphi,z,T)\frac{\partial V_1(r,\varphi,z,\tau)}{\partial z}d\tau\right]-\int_0^t[\alpha_{1V}+V_1(r,\varphi,z,\tau)]^2\times$$

$$\times k_{V,V}(r,\varphi,z,T)\,d\tau+\frac{\Omega}{r}\frac{\partial}{\partial r}\left\{r\cdot\int_0^t\frac{D_{VS}}{kT}\nabla_S\mu\,(r,\varphi,z,\tau)\int_0^L[\alpha_{2V}+V_1(r,\varphi,W,\tau)]\,dW\,d\tau\right\}+$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\left\{\int_0^t\frac{D_{VS}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L[\alpha_{2V}+V_1(r,\varphi,W,\tau)]\,dW\,d\tau\right\}-\int_0^t k_{I,V}(r,\varphi,z,T)\times$$

$$\times[\alpha_{1V}+I_1(r,\varphi,z,\tau)][\alpha_{1V}+V_1(r,\varphi,z,\tau)]\,d\tau$$

(2.d)

$$\Phi_{2I}(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t D_{\Phi_I}(r,\varphi,z,T)\frac{\partial\Phi_{1I}(r,\varphi,z,\tau)}{\partial r}d\tau\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[\int_0^t D_{\Phi_I}(r,\varphi,z,T)\times\right.$$

$$\left.\times\frac{\partial\Phi_{1I}(r,\varphi,z,\tau)}{\partial\varphi}d\,\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_I}(r,\varphi,z,T)\frac{\partial\Phi_{1I}(r,\varphi,z,\tau)}{\partial z}d\tau\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\int_0^t\frac{D_{\Phi_I S}}{kT}\times$$

$$\times\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2\Phi_I}+\Phi_{1I}(r,\varphi,W,\tau)\right]dW\,d\tau+\int_0^t k_{I,I}(r,\varphi,z,T)I^2(r,\varphi,z,\tau)d\tau+$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\int_0^t\frac{D_{\Phi_I S}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2\Phi_I}+\Phi_{1I}(r,\varphi,W,\tau)\right]dW\,d\tau+f_{\Phi_I}(r,\varphi,z)+$$

$$+\int_0^t k_I(r,\varphi,z,T)I(r,\varphi,z,\tau)d\tau,$$

$$\Phi_{2V}(r,\varphi,z,t)=\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot\int_0^t D_{\Phi_V}(r,\varphi,z,T)\frac{\partial\Phi_{1V}(r,\varphi,z,\tau)}{\partial r}d\tau\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[\int_0^t D_{\Phi_V}(r,\varphi,z,T)\times\right.$$

$$\left.\times\frac{\partial\Phi_{1V}(r,\varphi,z,\tau)}{\partial\varphi}d\,\tau\right]+\frac{\partial}{\partial z}\left[\int_0^t D_{\Phi_V}(r,\varphi,z,T)\frac{\partial\Phi_{1V}(r,\varphi,z,\tau)}{\partial z}d\tau\right]+\frac{\Omega}{r}\frac{\partial}{\partial r}\int_0^t\frac{D_{\Phi_V S}}{kT}\times$$

$$\times\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2\Phi_V}+\Phi_{1V}(r,\varphi,W,\tau)\right]dW\,d\tau+\int_0^t k_{V,V}(r,\varphi,z,T)V^2(r,\varphi,z,\tau)d\tau+$$

$$+\frac{\Omega}{r}\frac{\partial}{\partial\varphi}\int_0^t\frac{D_{\Phi_V S}}{kT}\nabla_S\mu(r,\varphi,z,\tau)\int_0^L\left[\alpha_{2\Phi_V}+\Phi_{1V}(r,\varphi,W,\tau)\right]dW\,d\tau+f_{\Phi_V}(r,\varphi,z)+$$

$$+\int_0^t k_V(r,\varphi,z,T)V(r,\varphi,z,\tau)d\tau.$$

Average values of the second-order approximations of the required approximations are obtained using the following standard relation [15–18].

$$\alpha_2=\frac{1}{\pi\Theta R^2 L}\int_0^{\Theta}\int_0^{R} r\int_0^{2\pi}\int_{-L}^{L}\left(\rho_2-\rho_1\right)d\,z\,d\,\varphi\,d\,r\,d\,t\,. \qquad \textbf{(8)}$$

Substitution of the *Relations (1d)* and *(2d)* into *Relation (8)* gives us the possibility to obtain relations for the required average values $\alpha_{2\rho}$

$$\alpha_{2C}=0,\ \alpha_{2\Phi I}=0,\ \alpha_{2\Phi V}=0,$$

$$\alpha_{2V}=\sqrt{\frac{\left(b_3+E\right)^2}{4b_4^2}-4\left(F+\frac{\Theta\,a_3F+2\pi\,L\,R\,\Theta^2 b_1}{b_4}\right)}-\frac{b_3+E}{4b_4},$$

$$\alpha_{2I}=\frac{C_V-\alpha_{2V}^2S_{VV00}-\alpha_{2V}\left(2S_{VV01}+S_{IV10}+2\pi\,L\,R\Theta\right)-S_{VV02}-S_{IV11}}{S_{IV01}+\alpha_{2V}S_{IV00}},$$

where $b_4=\frac{S_{IV00}^2S_{VV00}-S_{VV00}^2S_{II00}}{2\pi\,L\,R^2\Theta}$, $b_3=\left(S_{IV01}+2S_{II10}+S_{IV01}+2\pi\,L\,R^2\Theta\right)\frac{S_{IV00}S_{VV00}}{2\pi\,L\,R^2\Theta}-$

$$-\frac{S_{II00}S_{VV00}}{2\pi\,L\,R^2\Theta}\left(2S_{VV01}+S_{IV10}+2\pi\,L\,R^2\Theta\right)+S_{IV00}^2\frac{2S_{VV01}+S_{IV10}+2\pi\,L\,R^2\Theta}{2\pi\,L\,R^2\Theta}-\frac{S_{IV00}^2S_{IV10}}{6\pi^3L^3R^6\Theta^3},$$

$$b_2=S_{II00}S_{VV00}\frac{S_{VV02}+S_{IV11}+C_V}{2\pi\,L\,R^2\Theta}-\left(2\pi\,L\,R^2\Theta-2S_{VV01}+S_{IV10}\right)^2+\left(2\pi\,L\,R^2\Theta+2S_{II10}+S_{IV01}\right)\times$$

$$\times\frac{S_{IV01}S_{VV00}}{2\pi\,L\,R^2\Theta}+S_{IV00}\left(2\pi\,L\,R^2\Theta+S_{IV01}+2S_{II10}+2S_{IV01}\right)\frac{2S_{VV01}+2\pi\,L\,R^2\Theta+S_{IV10}}{2\pi\,L\,R^2\Theta}-S_{IV00}^2\times$$

$$\times\frac{C_V-S_{VV02}-S_{IV11}}{2\pi\,L\,R^2\Theta}+\frac{C_IS_{IV00}^2}{4\pi^2L^2\,R^2\Theta^2}-2\frac{S_{IV10}S_{IV00}S_{IV01}}{\pi\,L\,R^2\Theta},\ b_1=S_{II00}\frac{S_{IV11}+S_{VV02}+C_V}{2\pi\,L\,R^2\Theta}\left(2S_{VV01}+\right.$$

$$\left.+S_{IV10}+2\pi\,L\,R^2\right)+S_{IV01}\frac{2\pi\,L\,R^2\Theta+2S_{II10}+S_{IV01}}{2\pi\,L\,R^2\Theta}\left(2S_{VV01}+S_{IV10}+2\pi\,L\,R^2\Theta\right)+C_IS_{IV00}\times$$

$$\times 2S_{IV01}-S_{IV00}\frac{C_V-S_{VV02}-S_{IV11}}{2\pi\,L\,R^2\Theta}\left(3S_{IV01}+2S_{II10}+2\pi\,L\,R^2\Theta\right)-\frac{S_{IV10}S_{IV01}^2}{2\pi\,L\,R^2\Theta},\ b_0=S_{II00}\times$$

$$\times\frac{\left(S_{IV00}+S_{VV02}\right)^2}{2\pi\,L\,R^2\Theta}-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{2\pi\,L\,R^2\Theta}\left(2\pi\,L\,R^2\Theta+2S_{II10}+S_{IV01}\right)-\frac{C_V-S_{VV02}-S_{IV11}}{2\pi\,L\,R^2\Theta}\times$$

$$\times S_{IV01}\left(2\pi\,L\,R^2\Theta+2S_{II10}+S_{IV01}\right)+2C_IS_{IV01}^2,\ C_I=\frac{\alpha_{1I}\alpha_{1V}S_{IV00}+\alpha_{1I}^2S_{II00}-S_{II20}S_{II20}-S_{IV11}}{2\pi\,L\,R^2\Theta},$$

$$C_V=\alpha_{1I}\alpha_{1V}S_{IV00}-S_{IV11}+\alpha_{1V}^2S_{VV00}-S_{VV02},\ F=\frac{\Theta\,a_2}{6a_4}+\sqrt[3]{\sqrt{r^2+s^3}-r}-\sqrt[3]{\sqrt{r^2+s^3}+r},$$

$$E=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}},\ r=\frac{\Theta^3b_2}{12\,b_4^2}\left(2b_0-\pi\,L\,R^2\Theta\frac{b_1b_3}{b_4}\right)-b_0\frac{\Theta^2}{8b_4^2}\left(4\Theta\,b_2-\Theta^2\frac{b_3^2}{b_4}\right)-$$

$$-\frac{\Theta^3b_2^3}{54b_4^3}-4\pi^2L^2R^4\frac{\Theta^4b_1^2}{8b_4^2}-4\pi^2L^2R^4\frac{\Theta^4b_1^2}{8b_4^2},\ s=\frac{\Theta^2}{6b_4^2}\left(2b_0b_4-\pi\,L\,R^2\Theta\,b_1b_3\right)-\frac{\Theta\,b_2}{18b_4}.$$

Further, we calculate solutions to Eq. (6), i.e., the components of the displacement vector. To calculate the first-order approximations of the considered components in the framework of the method of averaging of function corrections, we replace the required functions in the right-hand sides of the equations by their not yet known average values α_i. The substitution leads to the following result:

$$\rho(z)\frac{\partial^2 u_{1x}(r,\varphi,z,t)}{\partial t^2} = -K(z)\frac{\beta(z)}{r}\frac{\partial\left[r\cdot T(r,\varphi,z,t)\right]}{\partial r},$$

$$\rho(z)\frac{\partial^2 u_{1y}(r,\varphi,z,t)}{\partial t^2} = -K(z)\frac{\beta(z)}{r}\frac{\partial T(r,\varphi,z,t)}{\partial \varphi},$$

$$\rho(z)\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial t^2} = -K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial z}.$$

Integration of the left and the right sides of the above relations over time t leads to the following result:

$$u_{1r}(r,\varphi,z,t) = K(z)\frac{\beta(z)}{r\cdot\rho(z)}\frac{\partial}{\partial r}\left[r\cdot\int_0^t\int_0^{\vartheta} T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right] -$$

$$-K(z)\frac{\beta(z)}{r\cdot\rho(z)}\frac{\partial}{\partial r}\left[r\cdot\int_0^t\int_0^{\vartheta} T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right] + u_{0r},$$

$$u_{1\varphi}(r,\varphi,z,t) = K(z)\frac{\beta(z)}{r\cdot\rho(z)}\frac{\partial}{\partial \varphi}\int_0^t\int_0^{\vartheta} T(r,\varphi,z,\tau)\,d\tau\,d\vartheta -$$

$$-K(z)\frac{\beta(z)}{r\cdot\rho(z)}\frac{\partial}{\partial \varphi}\int_0^{\infty}\int_0^{\vartheta} T(r,\varphi,z,\tau)\,d\tau\,d\vartheta + u_{0\varphi},$$

$$u_{1z}(r,\varphi,z,t) = K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^t\int_0^{\vartheta} T(r,\varphi,z,\tau)\,d\tau\,d\vartheta -$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta} T(r,\varphi,z,\tau)\,d\tau\,d\vartheta + u_{0z}.$$

Approximations of the second and higher orders of components of the displacement vector could be determined by using standard replacement of the required components in the following sums $\alpha_i + u_i(r,\varphi,z,t)$ [15–18]. The replacement leads to the following result:

$$\rho(z)\frac{\partial^2 u_{2r}(r,\varphi,z,t)}{\partial t^2} = \frac{1}{r}\left\{K(z)+\frac{5E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial u_{1r}(r,\varphi,z,t)}{\partial r}\right] - \frac{1}{r^2}\left\{\frac{E(z)}{3\left[1+\sigma(z)\right]} -\right.$$

$$-K(z)\Bigg\}\frac{\partial^2\left[r\cdot u_{1\varphi}(r,\varphi,z,t)\right]}{\partial r\,\partial\varphi}+\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{1}{r^2}\frac{\partial^2 u_{1y}(r,\varphi,z,t)}{\partial\varphi^2}+\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial z^2}\right]+$$

$$+\frac{1}{r}\left\{K(z)+\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2\left[r\cdot u_{1z}(r,\varphi,z,t)\right]}{\partial r\,\partial z}-K(z)\beta(z)\frac{\partial\left[r\cdot T_{1z}(r,\varphi,z,t)\right]}{\partial r},$$

$$\rho(z)\frac{\partial^2 u_{2\varphi}(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_{1\varphi}(r,\varphi,z,t)}{\partial r}\right]+\left[\frac{\partial^2 r\cdot u_{1r}(r,\varphi,z,t)}{\partial r\,\partial\varphi}\right]\right\}-$$

$$\rho(z)\frac{\partial^2 u_{2\varphi}(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_{1\varphi}(r,\varphi,z,t)}{\partial r}\right]+\left[\frac{\partial^2 r\cdot u_{1r}(r,\varphi,z,t)}{\partial r\,\partial\varphi}\right]\right\}-$$

$$-K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial y}+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2\left[1+\sigma(z)\right]}\left[\frac{\partial u_{1\varphi}(r,\varphi,z,t)}{\partial z}+\frac{1}{r}\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial\varphi}\right]\right\}+$$

$$+\frac{1}{r^2}\left\{\frac{5E(z)}{12\left[1+\sigma(z)\right]}+K(z)\right\}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi^2}+\frac{1}{r}\left\{K(z)-\frac{E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi\,\partial z}+$$

$$+\frac{K(z)}{r}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial r\,\partial\varphi},$$

$$\rho(z)\frac{\partial^2 u_{2z}(r,\varphi,z,t)}{\partial t^2}=\frac{E(z)}{2\left[1+\sigma(z)\right]}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 u_{1z}(r,\varphi,z,t)}{\partial\varphi^2}+\right.$$

$$\left.+\frac{\partial^2\left[r\cdot u_{1x}(r,\varphi,z,t)\right]}{\partial r\,\partial z}+\frac{1}{r}\frac{\partial^2 u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi\,\partial z}\right\}+\frac{\partial}{\partial z}\left(\left\{\frac{1}{r}\frac{\partial\left[r\cdot u_{1r}(r,\varphi,z,t)\right]}{\partial r}+\frac{1}{r}\frac{\partial u_{1\varphi}(r,\varphi,z,t)}{\partial\varphi}+\right.\right.$$

$$\left.\left.+\frac{\partial u_{1r}(r,\varphi,z,t)}{\partial z}\right\}\right)+\frac{1}{6}\frac{\partial}{\partial z}\left(\left\{6\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial z}-\frac{1}{r}\frac{\partial\left[r\cdot u_{1r}(r,\varphi,z,t)\right]}{\partial r}-\frac{1}{r}\frac{\partial u_{1y}(r,\varphi,z,t)}{\partial\varphi}-\right.\right.$$

$$\left.\left.-\frac{\partial u_{1z}(r,\varphi,z,t)}{\partial z}\right\}\frac{E(z)}{1+\sigma(z)}\right)-K(z)\beta(z)\frac{\partial T(r,\varphi,z,t)}{\partial z}.$$

Integration of the left and right sides of the above relations over time t leads to the following result:

$$u_{2r}(r,\varphi,z,t)=\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6\left[1+\sigma(z)\right]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^t\int_0^{\vartheta}u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{1}{\rho(z)}\times$$

$$\times\left\{K(z)-\frac{E(z)}{3\left[1+\sigma(z)\right]}\right\}\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^t\int_0^{\vartheta}u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{E(z)}{2r^2\rho(z)\left[1+\sigma(z)\right]}\times$$

$$\times\left[\frac{\partial^2}{\partial\varphi^2}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\left\{K(z)+\frac{E(z)}{3\,[1+\sigma(z)]}\right\}\times$$

$$\times\frac{1}{\rho(z)}\frac{\partial^2}{\partial r\,\partial z}\left[r\int_0^t\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial r}\left[r\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\frac{1}{\rho(z)}\times$$

$$\times\left\{K(z)+\frac{5E(z)}{6\,[1+\sigma(z)]}\right\}\frac{\partial}{\partial r}\left[r\frac{\partial^2}{\partial r^2}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]-\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times$$

$$\times\frac{1}{\rho(z)}\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^\infty\int_0^\vartheta u_{1y}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]-\frac{E(z)}{2\,\rho(z)}\left[\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)d\tau\,d\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial z^2}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]\frac{1}{1+\sigma(z)}-\frac{\partial^2}{\partial r\,\partial z}\left[r\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]\left\{\frac{E(z)}{3\,[1+\sigma(z)]}+\right.$$

$$\left.+K(z)\right\}\frac{1}{\rho(z)}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial r}\left[r\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+u_{0r},$$

$$u_{2\varphi}(r,\varphi,z,t)=\left\{\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]+\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]\right\}\times$$

$$\times\frac{E(z)}{2r\rho(z)[1+\sigma(z)]}+\frac{1}{r^2\rho(z)}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\frac{\partial^2}{\partial\varphi^2}\int_0^t\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau\,d\vartheta+$$

$$+\frac{K(z)}{r\,\rho(z)}\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^t\int_0^\vartheta u_{1y}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]-K(z)\frac{\beta(z)}{\rho(z)}\int_0^t\int_0^\vartheta T(r,\varphi,z,\tau)d\tau\,d\vartheta+$$

$$+\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^t\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]\right\}+$$

$$+\frac{1}{r\,\rho(z)}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial\varphi\,\partial z}\int_0^t\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)d\tau\,d\vartheta-\frac{E(z)}{2r\,\rho(z)\,[1+\sigma(z)]}\times$$

$$\times\left\{\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]+\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]\right\}-K(z)\frac{\beta(z)}{\rho(z)}\times$$

$$\times\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)d\tau\,d\vartheta-\frac{1}{r^2\rho(z)}\frac{\partial^2}{\partial\varphi^2}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\left\{K(z)+\frac{5E(z)}{12[1+\sigma(z)]}\right\}-$$

$$-\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)d\tau\,d\vartheta+\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)d\tau\,d\vartheta\right]\right\}-$$

$$-\frac{K(z)}{r\rho(z)}\frac{\partial^2}{\partial r\,\partial\varphi}\left[r\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\frac{1}{r\,\rho(z)}\frac{\partial^2}{\partial\varphi\,\partial z}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\times$$

$$\times\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}+u_{0\varphi},$$

$$u_{2z}(r,\varphi,z,t)=\frac{E(z)}{2r\,\rho(z)}\left\{\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{1}{r}\frac{\partial^2}{\partial\varphi^2}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial r\,\partial z}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{\partial^2}{\partial\varphi\,\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right\}\frac{1}{1+\sigma(z)}+\frac{1}{r\,\rho(z)}\times$$

$$\times\frac{\partial}{\partial z}\left(K(z)\left\{\frac{\partial}{\partial r}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]+\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta+\right.\right.$$

$$\left.\left.+\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right\}\right)+\frac{1}{6\rho(z)}\frac{\partial}{\partial z}\left(\frac{E(z)}{1+\sigma(z)}\left\{6\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta-\right.\right.$$

$$-\frac{1}{r}\frac{\partial}{\partial r}\left[r\int_0^\infty\int_0^\vartheta u_{1r}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right]-\frac{1}{r}\frac{\partial}{\partial\varphi}\int_0^\infty\int_0^\vartheta u_{1\varphi}(r,\varphi,z,\tau)\,d\tau\,d\vartheta-$$

$$\left.\left.-\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(r,\varphi,z,\tau)\,d\tau\,d\vartheta\right\}\right)-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(r,\varphi,z,\tau)\,d\tau\,d\vartheta+u_{0z}.$$

Now we determine and analyze the spatio-temporal distribution of temperature. To analyze the distribution, we determine the solution of the second Fourier law [19].

$$c\frac{\partial T(r,\varphi,z,t)}{\partial t}=div\{\lambda\cdot grad[T(r,\varphi,z,t)]-\vec{v}(r,\varphi,z,t)\cdot c(T)\cdot T(r,\varphi,z,t)\cdot C(r,\varphi,z,t)\}+$$
$$+p(r,\varphi,z,t). \quad (9)$$

Here $\vec{v}$ is the speed of flow of mixture of gases-reagents (we consider gases- reagents as ideal gases); c is the heat capacity; T (r,φ,z,t) is the spatio-temporal distribution of temperature; p (r,φ,z,t) is the density of power in the system substrate-keeper of substrate; r, φ, z and t are the cylindrical coordinates and time; C (r,φ,z,t) is the spatio-temporal distribution of concentration of mixture of gases-reagents; λ is the heat conductivity. The following relation can determine the value of heat conductivity: $\lambda=\bar{v}\bar{l}c_v\rho/3$, where $\bar{v}$ is the speed of the gas molecules, $\bar{l}$ is the average free path of gas molecules between collisions, c_v is the specific heat at constant volume, and ρ is the density of gas.

To solve *Eq. (9),* we shall take into account the movement of the mixture of gases and the concentration of the mixture. We determine the fluid velocity and concentration by solving the Navier-Stokes equation and the second Fick's law, respectively. We also assume that the radius of the substrate keeper, R, is essentially larger than the thicknesses of the diffusion and near-boundary layers. We also assume the gas stream is laminar. In this situation, the appropriate equations could be written as

$$\frac{\partial \vec{v}}{\partial t}+\left(\vec{v}\cdot\nabla\right)\vec{v}=-\nabla\left(\frac{P}{\rho}\right)+\nu\,\Delta\vec{v}\,, \tag{10}$$

$$\frac{\partial C(r,\varphi,z,t)}{\partial t}=div\{D\cdot grad\left[C(r,\varphi,z,t)\right]-\vec{v}(r,\varphi,z,t)\cdot C(r,\varphi,z,t)\}, \tag{11}$$

where D is the diffusion coefficient of the mixture of gases-reagents; P is the pressure; ρ is the density; ν is the kinematic viscosity. Let us consider the regime of the limiting flow, in which all molecules of the deposit material approaching the disk deposit on the substrate; the flow is homogeneous and one-dimensional. In this case, boundary and initial conditions could be written as

$C(r,\varphi,-L,t) = C_0$, $C(r,-\varphi_1,z,t) = C(r,\varphi_1,z,t) = C(r,\pi-\varphi_1,z,t) = C(r,\pi+\varphi_1,z,t)$, $C(r,\varphi,z,0) =$
$C_0\delta(z+L)$, $C(0,\varphi,z,t) \neq \infty$, $C(r,\varphi,z,t)\big|_S=0$, $T(r,-\varphi_1,z,t) = T(r,\varphi_1,z,t) = T(r,\pi-\varphi_1,z,t) =$
$T(r,\pi+\varphi_1,z,t)$, $\left.\frac{\partial C(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0$, $-\lambda\left.\frac{\partial T(r,\varphi,z,t)}{\partial r}\right|_S=\sigma T^4(R,\varphi,z,t)$, $T(r,\varphi,z,0) =$ **(12)**
$=T_r$, $\left.\frac{\partial T(r,\varphi,z,t)}{\partial \varphi}\right|_{\varphi=0}=\left.\frac{\partial T(r,\varphi,z,t)}{\partial \varphi}\right|_{\varphi=2\pi}$,
$-\lambda\left.\frac{\partial T(r,\varphi,z,t)}{\partial z}\right|_S=\sigma T^4(r,\varphi,-L,t)$,

$T(0,\varphi,z,t) \neq \infty$, $\left.\frac{\partial v_r(r,\varphi,z,t)}{\partial r}\right|_{r=0}=0$, $\left.\frac{\partial v_r(r,\varphi,z,t)}{\partial r}\right|_{r=R}=0$, $v_r(r,-\varphi_1,z,t) = v_r(r,\varphi_1,z,t) =$
$v_r(r,\pi-\varphi_1,z,t) = v_r(r,\pi+\varphi_1,z,t)$, $v_\varphi(r,-\varphi_1,z,t) = v_\varphi(r,\varphi_1,z,t) = v_\varphi(r,\pi-\varphi_1,z,t) = v_\varphi(r,\pi+\varphi_1,z,t)$,
$v_z(r,-\varphi_1,z,t) = v_z(r,\varphi_1,z,t) = v_z(r,\pi-\varphi_1,z,t) = v_z(r,\pi+\varphi_1,z,t)$, $v_r(r,\varphi,-L,t) = 0$, $v_r(r,\varphi,L,t) = 0$,
$v_r(0,\varphi,z,t) \neq \infty$, $v_z(r,\varphi,-L,t) = V_0$, $v_z(r\pm d_2/2,\varphi,z\in[-d_2/2,d_2/2],0) = \omega\cdot z\cdot\cos\psi\ \mathrm{tg}(\varphi_1)$,
$v_\varphi(r,\varphi,L,t) = 0$, $v_\varphi(0,\varphi,z,t) \neq \infty$, $v_z(r,\varphi,0,t) = 0$, $v_z(r,\varphi,L,t) = V_0$, $v_z(r,\varphi,L,t) = V_0$,
$v_z(0,\varphi,z,t) \neq \infty$, $v_r(r,\varphi,z,0) = 0$, $v_\varphi(r,\varphi,z,0) = 0$,

where $\sigma = 5{,}67\cdot10^{-8}$ W·m^{-2}·K^{-4}, T_r is the room temperature, ω is the frequency of rotation of the substrate.

Equations for components of velocity of flow with account cylindrical system of coordinate could be written as

$$\frac{\partial \upsilon_r}{\partial t} = \nu \left\{ \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial \upsilon_r(r,\varphi,z,t)}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 \upsilon_r(r,\varphi,z,t)}{\partial \varphi^2} + \frac{\partial^2 \upsilon_r(r,\varphi,z,t)}{\partial z^2}\right\} -$$

(10.a)

$$-\upsilon_r \frac{\partial \upsilon_r}{\partial r} - \frac{\upsilon_\varphi}{r}\frac{\partial \upsilon_\varphi}{\partial \varphi} - \upsilon_z \frac{\partial \upsilon_z}{\partial z} - \frac{\partial}{\partial r}\left(\frac{P}{\rho}\right),$$

$$\frac{\partial \upsilon_r}{\partial t} = \nu \left\{ \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial \upsilon_\varphi(r,\varphi,z,t)}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 \upsilon_\varphi(r,\varphi,z,t)}{\partial \varphi^2} + \frac{\partial^2 \upsilon_\varphi(r,\varphi,z,t)}{\partial z^2}\right\} -$$

(10.b)

$$-\upsilon_r \frac{\partial \upsilon_r}{\partial r} - \frac{\upsilon_\varphi}{r}\frac{\partial \upsilon_\varphi}{\partial \varphi} - \upsilon_z \frac{\partial \upsilon_z}{\partial z} - \frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{P}{\rho}\right),$$

$$\frac{\partial \upsilon_z}{\partial t} = \nu \left\{ \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial \upsilon_z(r,\varphi,z,t)}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 \upsilon_z(r,\varphi,z,t)}{\partial \varphi^2} + \frac{\partial^2 \upsilon_z(r,\varphi,z,t)}{\partial z^2}\right\} -$$

(10.c)

$$-\upsilon_r \frac{\partial \upsilon_r}{\partial r} - \frac{\upsilon_\varphi}{r}\frac{\partial \upsilon_\varphi}{\partial \varphi} - \upsilon_z \frac{\partial \upsilon_z}{\partial z} - \frac{\partial}{\partial z}\left(\frac{P}{\rho}\right).$$

We determine the solution of this system of equations using the method of averaging of function corrections [15–18]. Within the framework of the approach, we first determine the first-order approximations of the components of the mixture's flow speed. To determine the first-order approximation, we replace the required functions with their average values vr((1r, v(((1(, vz((1z on the right-hand sides of the equations of *system (10)*. After the replacement and calculation of the required derivatives, we obtain equations for the first-order approximations of the components:

$$\frac{\partial v_{1r}}{\partial t} = -\frac{\partial}{\partial r}\left(\frac{P}{\rho}\right),\ \frac{\partial v_{1\varphi}}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{P}{\rho}\right),\ \frac{\partial v_{1z}}{\partial t} = -\frac{\partial}{\partial z}\left(\frac{P}{\rho}\right).$$

Integration of the left and the right sides with respect to time of the *Relations (12)* gives us the possibility to obtain the first-order approximations of the components of speed of flow in the final form.

$$v_{1r} = -\frac{\partial}{\partial r}\int_0^t \frac{P}{\rho}\,d\tau,\ v_{1\varphi} = -\frac{1}{r}\frac{\partial}{\partial \varphi}\int_0^t \frac{P}{\rho}\,d\tau,\ v_{1z} = -\frac{\partial}{\partial z}\int_0^t \frac{P}{\rho}\,d\tau. \quad \textbf{(13)}$$

The second-order approximations of components of speed of flow could be obtained by replacement of the required functions on the following sums $v_r \to \alpha_{2r}+v_{1r}$, $v_\varphi \to \alpha_{2\varphi}+ v_{2r}$, $v_z \to \alpha_{2z}+v_{2r}$. The average values α_{2r}, $\alpha_{2\varphi}$, α_{2z} are not yet known. Approximations for the components could be written as

$$\frac{\partial v_{2r}}{\partial t} = \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_{1r}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_{1r}}{\partial \varphi^2} + \frac{\partial^2 v_{1r}}{\partial z^2}\right] - \frac{\partial}{\partial r}\left(\frac{P}{\rho}\right) -$$

(14.a)

$$-\left(\alpha_{2r}+v_{1r}\right)\frac{\partial v_{1r}}{\partial r} - \frac{\left(\alpha_{2\varphi}+v_{1\varphi}\right)}{r}\frac{\partial v_{1r}}{\partial \varphi} - \left(\alpha_{2z}+v_{1z}\right)\frac{\partial v_{1r}}{\partial z},$$

$$\frac{\partial v_{2\varphi}}{\partial t} = \nu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_{1\varphi}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_{1\varphi}}{\partial \varphi^2} + \frac{\partial^2 v_{1\varphi}}{\partial z^2}\right] - \frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{P}{\rho}\right) - $$
$$- \left(\alpha_{2r} + v_{1r}\right)\frac{\partial v_{1\varphi}}{\partial r} - \frac{\left(\alpha_{2\varphi} + v_{1\varphi}\right)}{r}\frac{\partial v_{1\varphi}}{\partial \varphi} - \left(\alpha_{2z} + v_{1z}\right)\frac{\partial v_{1\varphi}}{\partial z}, \tag{14.b}$$

$$\frac{\partial v_{2z}}{\partial t} = \nu \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_{1z}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_{1z}}{\partial \varphi^2} + \frac{\partial^2 v_{1z}}{\partial z^2}\right] - \frac{\partial}{\partial z}\left(\frac{P}{\rho}\right) - $$
$$- \left(\alpha_{2r} + v_{1r}\right)\frac{\partial v_{1z}}{\partial r} - \frac{\left(\alpha_{2\varphi} + v_{1\varphi}\right)}{r}\frac{\partial v_{1z}}{\partial \varphi} - \left(\alpha_{2z} + v_{1z}\right)\frac{\partial v_{1z}}{\partial z}. \tag{14.c}$$

Integration of the above equations over time leads to the following result:

$$v_{2r} = \nu \int_0^t \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_{1r}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_{1r}}{\partial \varphi^2} + \frac{\partial^2 v_{1r}}{\partial z^2}\right] d\tau - \frac{\partial}{\partial r}\left(\int_0^t \frac{P}{\rho} d\tau\right) - $$
$$- \int_0^t \left(\alpha_{2r} + v_{1r}\right)\frac{\partial v_{1r}}{\partial r} d\tau - \int_0^t \frac{\left(\alpha_{2\varphi} + v_{1\varphi}\right)}{r}\frac{\partial v_{1r}}{\partial \varphi} d\tau - \int_0^t \left(\alpha_{2z} + v_{1z}\right)\frac{\partial v_{1r}}{\partial z} d\tau, \tag{14.d}$$

$$v_{2\varphi} = \nu \int_0^t \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_{1\varphi}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_{1\varphi}}{\partial \varphi^2} + \frac{\partial^2 v_{1\varphi}}{\partial z^2}\right] d\tau - \frac{1}{r}\frac{\partial}{\partial \varphi}\left(\int_0^t \frac{P}{\rho} d\tau\right) - $$
$$- \int_0^t \left(\alpha_{2r} + v_{1r}\right)\frac{\partial v_{1\varphi}}{\partial r} d\tau - \int_0^t \frac{\left(\alpha_{2\varphi} + v_{1\varphi}\right)}{r}\frac{\partial v_{1\varphi}}{\partial \varphi} d\tau - \int_0^t \left(\alpha_{2z} + v_{1z}\right)\frac{\partial v_{1\varphi}}{\partial z} d\tau, \tag{14.e}$$

$$v_{2z} = V_0 + \nu \int_0^t \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_{1z}}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_{1z}}{\partial \varphi^2} + \frac{\partial^2 v_{1z}}{\partial z^2}\right] d\tau - \frac{\partial}{\partial z}\left(\int_0^t \frac{P}{\rho} d\tau\right) - $$
$$- \int_0^t \left(\alpha_{2r} + v_{1r}\right)\frac{\partial v_{1z}}{\partial r} d\tau - \int_0^t \frac{\left(\alpha_{2\varphi} + v_{1\varphi}\right)}{r}\frac{\partial v_{1z}}{\partial \varphi} d\tau - \int_0^t \left(\alpha_{2z} + v_{1z}\right)\frac{\partial v_{1z}}{\partial z} d\tau. \tag{14.f}$$

Further, we determine average values α_{2r}, $\alpha_{2\varphi}$, α_{2z}. The average values have been calculated by the following relations [15–18].

$$\alpha_{2r} = \frac{1}{\pi\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\left(v_{2r}-v_{1r}\right)d\,z\,d\,\varphi\,d\,r\,d\,t\,,$$

$$\alpha_{2\varphi} = \frac{1}{\pi\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\left(v_{2\varphi}-v_{1\varphi}\right)d\,z\,d\,\varphi\,d\,r\,d\,t\,, \tag{15}$$

$$\alpha_{2z} = \frac{1}{\pi\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\left(v_{2z}-v_{1z}\right)d\,z\,d\,\varphi\,d\,r\,d\,t\,,$$

where Θ is the continuous movement of a mixture of gases through the considered horizontal reactor. Substitution of the first- and the second-order approximations of the required components of speed into the *Relations (15)* gives us the possibility to obtain a system of equations to determine required average values:

$$\begin{cases} A_1\alpha_{2r} + B_1\alpha_{2\varphi} + C_1\alpha_{2z} = D_1 \\ A_2\alpha_{2r} + B_2\alpha_{2\varphi} + C_2\alpha_{2z} = D_2 \\ A_3\alpha_{2r} + B_3\alpha_{2\varphi} + C_3\alpha_{2z} = D_3 \end{cases} \tag{16}$$

where $A_1 = 1 + \int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\frac{\partial v_{1r}}{\partial r}d\,z\,d\,\varphi\,d\,r\,d\,t$, $B_1 = \int_0^\Theta(\Theta-t)\int_0^R\int_0^{2\pi}\int_{-L}^{L}\frac{\partial v_{1r}}{\partial \varphi}d\,z\,d\,\varphi\,d\,r\,d\,t$,

$$C_1 = C_2 = \frac{\pi}{2}\Theta^2R^2V_0,\; D_1 = \nu\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\, v_{1r}}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_{1r}}{\partial\varphi^2}+\frac{\partial^2 v_{1r}}{\partial z^2}\right]d\,z\,d\,\varphi\,d\,r\,(\Theta-$$

$$-t)\,d\,t - \frac{\pi}{8}\Theta^2R^2V_0^2 - \int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}v_{1r}\frac{\partial v_{1r}}{\partial r}d\,z\,d\,\varphi\,d\,r\,d\,t - \int_0^\Theta\int_0^R\int_0^{2\pi}\int_{-L}^{L}v_{1\varphi}\frac{\partial v_{1r}}{\partial\varphi}d\,z\,d\,\varphi\,d\,r\,\times$$

$$\times(\Theta-t)d\,t,\; A_2 = \int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\frac{\partial v_{1r}}{\partial r}d\,z\,d\,\varphi\,d\,r\,d\,t,\; B_2 = 1+\int_0^\Theta\int_0^R\int_0^{2\pi}\int_{-L}^{L}\frac{\partial v_{1r}}{\partial\varphi}d\,z\,d\,\varphi\,d\,r(\Theta-$$

$$-t)d\,t,\; D_2 = \nu\int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\, v_{1\varphi}}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_{1\varphi}}{\partial\varphi^2}+\frac{\partial^2 v_{1\varphi}}{\partial z^2}\right]d\,z\,d\,\varphi\,d\,r\,d\,t - \int_0^\Theta(\Theta-t)\times$$

$$\times\int_0^R r\int_0^{2\pi}\int_{-L}^{L}v_{1r}\frac{\partial v_{1r}}{\partial r}d\,z\,d\,\varphi\,d\,r\,d\,t - \int_0^\Theta(\Theta-t)\int_0^R\int_0^{2\pi}\int_{-L}^{L}v_{1\varphi}\frac{\partial v_{1r}}{\partial\varphi}d\,z\,d\,\varphi\,d\,r\,d\,t - \frac{\pi}{8}\Theta^2R^2V_0^2,\; C_3 = 1+$$

$$+\frac{\pi}{2}\Theta^2R^2V_0, A_3 = \int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\frac{\partial v_{1z}}{\partial r}d\,z\,d\,\varphi\,d\,r\,d\,t,\; B_3 = \int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\frac{\partial v_{1z}}{\partial\varphi}d\,z\,d\,\varphi\,d\,r\,d\,t,$$

$$D_3 = \nu\int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\, v_{1z}}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_{1z}}{\partial\varphi^2}+\frac{\partial^2 v_{1z}}{\partial z^2}\right]d\,z\,d\,\varphi\,d\,r\,d\,t - \frac{\pi}{8}\Theta^2R^2V_0^2 - \int_0^\Theta(\Theta-$$

$$-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}v_{1r}\frac{\partial v_{1z}}{\partial r}d\,z\,d\,\varphi\,d\,r\,d\,t - \int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}v_{1\varphi}\frac{\partial v_{1z}}{\partial\varphi}d\,z\,d\,\varphi\,d\,r\,d\,t\,.$$

The solution of the above system of equations could be determined by standard approaches [20] and could be written as

$$\alpha_{2r} = \Delta_r/\Delta,\ \alpha_{2\varphi} = \Delta_\varphi/\Delta,\ \alpha_{2z} = \Delta_z/\Delta, \tag{17}$$

where $\Delta = A_1(B_2C_3 - B_3C_2) - B_1(A_2C_3 - A_3C_2) + C_1(A_2B_3 - A_3B_2)$, $\Delta_r = D_1(B_2C_3 - B_3C_2) -$
$- B_1(D_2C_3 - D_3C_2) + C_1(D_2B_3 - D_3B_2)$, $\Delta_\varphi = D_1(B_2C_3 - B_3C_2) - B_1(D_2C_3 - D_3C_2) + C_1 \times$
$\times (D_2B_3 - D_3B_2)$, $\Delta_z = A_1(B_2D_3 - B_3D_2) - B_1(A_2D_3 - A_3D_2) + D_1(A_2B_3 - A_3B_2)$.

In this section, we obtained components of the velocity of the stream of a gas-phase mixture used for the growth of a heterostructure, and the gas carrier in the second-order approximation framework for averaging-function corrections. Usually, the second-order approximation is sufficient for a qualitative analysis of the solution and for obtaining some quantitative results. Let us rewrite *Eqs. (10)* and *(11)* by using a cylindrical coordinate system:

$$c\frac{\partial T(r,\varphi,z,t)}{\partial t} = \lambda\frac{\partial^2 T(r,\varphi,z,t)}{\partial r^2} + \frac{\lambda}{r^2}\frac{\partial^2 T(r,\varphi,z,t)}{\partial \varphi^2} + \lambda\frac{\partial^2 T(r,\varphi,z,t)}{\partial z^2} -$$
$$- c\cdot\frac{\partial}{\partial r}\left[v_r(r,\varphi,z,t)\cdot C(r,\varphi,z,t)\cdot T(r,\varphi,z,t)\right] - \frac{c}{r}\frac{\partial}{\partial \varphi}\left[v_\varphi(r,\varphi,z,t)\cdot C(r,\varphi,z,t)\cdot\right. \tag{18}$$
$$\left.\cdot T(r,\varphi,z,t)\right] - c\cdot\frac{\partial}{\partial z}\left[v_z(r,\varphi,z,t)\cdot C(r,\varphi,z,t)\cdot T(r,\varphi,z,t)\right] + p(r,\varphi,z,t),$$

$$\frac{\partial C(r,\varphi,z,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\,D\frac{\partial C(r,\varphi,z,t)}{\partial r}\right] - \frac{1}{r}\frac{\partial}{\partial r}\left[r\,C(r,\varphi,z,t)v_r(r,\varphi,z,t)\right] +$$
$$+ \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left[D\frac{\partial C(r,\varphi,z,t)}{\partial \varphi}\right] - \frac{1}{r}\frac{\partial}{\partial \varphi}\left[C(r,\varphi,z,t)v_\varphi(r,\varphi,z,t)\right] + \tag{19}$$
$$+ \frac{\partial}{\partial z}\left[D\frac{\partial C(r,\varphi,z,t)}{\partial z}\right] - \frac{\partial}{\partial z}\left[C(r,\varphi,z,t)v_z(r,\varphi,z,t)\right].$$

In this section, we calculate the components of the speed of the gas reagents used to grow an epitaxial layer and the gas carrier, using a second-order approximation framework for averaging function corrections. Usually, the second-order approximation is sufficient for qualitative analysis and for obtaining some quantitative results. The results of the analytical calculation have been checked against those of the numerical simulation.

To determine the spatio-temporal distributions of temperature and the gas mixture concentration, we used the method of averaging function corrections. To determine the first- order approximations of the required functions, we replace them with their not-yet-known average values α_{1T} and α_{1C}) on the right sides of the above equations. Further, we used the recently considered algorithm to obtain the first-order approximations of temperature and concentration of gas-reagents:

$$T_1(r,\varphi,z,t)=T_r+\int_0^t \frac{p(r,\varphi,z,\tau)}{c}d\tau-\alpha_{1T}\alpha_{1C}\int_0^t \frac{\partial v_r(r,\varphi,z,\tau)}{\partial r}d\tau-$$
$$-\frac{\alpha_{1T}\alpha_{1C}}{r}\int_0^t \frac{\partial v_\varphi(r,\varphi,z,\tau)}{\partial \varphi}d\tau-\alpha_{1T}\alpha_{1C}\int_0^t \frac{\partial v_z(r,\varphi,z,\tau)}{\partial z}d\tau, \quad (20)$$

$$C_1(r,\varphi,z,t)=-\alpha_{1T}\alpha_{1C}\int_0^t \frac{\partial v_r(r,\varphi,z,\tau)}{\partial r}d\tau-\frac{\alpha_{1C}}{r}\int_0^t \frac{\partial[r\,v_r(r,\varphi,z,\tau)]}{\partial r}d\tau-$$
$$-\frac{\alpha_{1C}}{r}\int_0^t \frac{\partial v_\varphi(r,\varphi,z,\tau)}{\partial \varphi}d\tau-\alpha_{1C}\int_0^t \frac{\partial v_z(r,\varphi,z,\tau)}{\partial z}d\tau. \quad (21)$$

The standard relations could determine the above not-yet-known average values.

$$\alpha_{1T}=\frac{1}{\pi\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L} T_1(r,\varphi,z,\tau)\,dz\,d\varphi\,dr\,dt,$$
$$\alpha_{1C}=\frac{1}{\pi\Theta R^2 L}\int_0^\Theta\int_0^R r\int_0^{2\pi}\int_{-L}^{L} C_1(r,\varphi,z,\tau)\,dz\,d\varphi\,dr\,dt. \quad (22)$$

Substitution of the first-order approximations for the temperature and gas concentration in Relation (22) yields the following results [20].

$$\alpha_{1C}=C_0\Big/L\cdot\left[1+\frac{1}{\pi\Theta R L}\int_0^\Theta(\Theta-t)\int_0^{2\pi}\int_{-L}^{L} v_r(R,\varphi,z,t)\,dz\,d\varphi\,dt+\frac{\Theta V_0}{R L}\right],$$

$$\alpha_{1T}=\left[T_r+\frac{1}{\pi\Theta R^2 L}\int_0^\Theta(\Theta-t)\int_0^R r\int_0^{2\pi}\int_{-L}^{L}\frac{p(r,\varphi,z,t)}{c}dz\,d\varphi\,dr\,dt\right]\left\{1+\frac{C_0}{\pi\Theta R L^2}\left[\int_0^\Theta(\Theta-t)\times\right.\right.$$
$$\times\int_0^{2\pi}\int_{-L}^{L} v_r(R,\varphi,z,\tau)\,dz\,d\varphi\,dt-\frac{1}{\pi\Theta R^2}\int_0^\Theta(\Theta-t)\int_0^R\int_0^{2\pi}\int_{-L}^{L} v_r(r,\varphi,z,\tau)\,dz\,d\varphi\,dr\,dt+\left.\frac{V_0}{2}\right]\times$$
$$\left.\times\left[1+\frac{1}{\pi\Theta R L}\int_0^\Theta(\Theta-t)\int_0^{2\pi}\int_{-L}^{L} v_r(R,\varphi,z,t)\,dz\,d\varphi\,dt+\frac{\Theta V_0}{R L}\right]^{-1}\right\}.$$

The second-order approximations of temperature and concentration of gases- reagents we determine frame the method of averaging of function corrections [15–18], i.e. by replacement of the required functions in right sides of *Equations (18)* and *(19)* on the following sums $T\to\alpha_{2T}+T_1$, $C\to\alpha_{2C}+C_1$. In this case, the second-order approximations of the above required functions could be written as

$$c \cdot T_2(r,\varphi,z,t) = \lambda \int_0^t \frac{\partial^2 T_1(r,\varphi,z,\tau)}{\partial r^2} d\tau + \lambda \frac{1}{r^2} \int_0^t \frac{\partial^2 T_1(r,\varphi,z,\tau)}{\partial \varphi^2} d\tau + \lambda \int_0^t \frac{\partial^2 T_1(r,\varphi,z,\tau)}{\partial z^2} d\tau +$$
$$+ \int_0^t p(r,\varphi,z,\tau) d\tau - c \cdot \frac{\partial}{\partial r} \int_0^t \{v_r(r,\varphi,z,\tau) \cdot [\alpha_{2C} + C_1(r,\varphi,z,\tau)] \cdot [\alpha_{2T} + T_1(r,\varphi,z,\tau)]\} d\tau +$$
$$+ T_r - \frac{c}{r} \frac{\partial}{\partial \varphi} \int_0^t \{v_\varphi(r,\varphi,z,\tau) \cdot [\alpha_{2C} + C_1(r,\varphi,z,\tau)] \cdot [\alpha_{2T} + T_1(r,\varphi,z,\tau)]\} d\tau - \quad (23)$$
$$- c \cdot \frac{\partial}{\partial z} \int_0^t \{v_r(r,\varphi,z,\tau) \cdot [\alpha_{2C} + C_1(r,\varphi,z,\tau)] \cdot [\alpha_{2T} + T_1(r,\varphi,z,\tau)]\} d\tau,$$

$$C_2(r,\varphi,z,t) = \frac{1}{r} \frac{\partial}{\partial r} \int_0^t r D \frac{\partial C_1(r,\varphi,z,\tau)}{\partial r} d\tau + \frac{1}{r^2} \frac{\partial}{\partial \varphi} \int_0^t D \frac{\partial C_1(r,\varphi,z,\tau)}{\partial \varphi} d\tau + \frac{\partial}{\partial z} \int_0^t D \times$$
$$\times \frac{\partial C_1(r,\varphi,z,\tau)}{\partial z} d\tau - \frac{1}{r} \frac{\partial}{\partial r} \left\{ r \int_0^t [\alpha_{2C} + C_1(r,\varphi,z,\tau)] \cdot v_r(r,\varphi,z,\tau) d\tau \right\} - \frac{1}{r} \frac{\partial}{\partial \varphi} \int_0^t v_\varphi(r,\varphi,z,\tau) \cdot \quad (24)$$
$$\cdot [\alpha_{2C} + C_1(r,\varphi,z,\tau)] d\tau - \frac{\partial}{\partial z} \int_0^t [\alpha_{2C} + C_1(r,\varphi,z,\tau)] \cdot v_z(r,\varphi,z,\tau) d\tau + C_0 \delta(z+L).$$

Average values of the second-order approximations of the temperature and concentration of the mixture (2T and 2C) have been calculated using the following standard relations.

$$\alpha_{2T} = \frac{1}{\pi \Theta R^2 L} \int_0^\Theta \int_0^R r \int_0^{2\pi} \int_{-L}^L (T_2 - T_1) \, dz \, d\varphi \, dr \, dt,$$
$$\alpha_{2C} = \frac{1}{\pi \Theta R^2 L} \int_0^\Theta \int_0^R r \int_0^{2\pi} \int_{-L}^L (C_2 - C_1) \, dz \, d\varphi \, dr \, dt. \quad (25)$$

Substitution of the first- and the second-order approximations of temperature and concentration of mixture into *Relations (24)* gives us the possibility to obtain equations to determine required average values

$$\alpha_{2T}=\left(\frac{\lambda\sigma}{c\pi\Theta RL}\int_0^{\Theta}(\Theta-t)\int_0^{2\pi}\int_{-L}^{L}T^4(R,\varphi,z,t)\,d\,z\,d\,\varphi\,d\,t-\int_0^{\Theta}(\Theta-t)\int_0^{2\pi}\int_{-L}^{L}T_1(R,\varphi,z,t)\,d\,z\,d\,\varphi\,d\,t\times\right.$$

$$\times\frac{\lambda}{c\pi\Theta R^2L}+\frac{\lambda}{c\pi\Theta R^2L}\int_0^{\Theta}(\Theta-t)\int_0^{2\pi}\int_{-L}^{L}T_1(0,\varphi,z,t)\,d\,z\,d\,\varphi\,d\,t-\int_0^{\Theta}(\Theta-t)\int_0^{2\pi}\int_{-L}^{L}v_r(R,\varphi,z,t)\times$$

$$\times\left\{\left[\alpha_{2C}+C_1(R,\varphi,z,t)\right]T_1(R,\varphi,z,t)-\alpha_{1T}\alpha_{1C}\right\}d\,z\,d\,\varphi\,d\,t\frac{1}{\pi\Theta RL}-\frac{1}{\pi\Theta R^2L}\int_0^{\Theta}(\Theta-t)\times$$

$$\times\int_0^{R}\int_0^{2\pi}\int_{-L}^{L}v_r(r,\varphi,z,\tau)\left\{T_1(r,\varphi,z,t)\left[\alpha_{2C}+C_1(r,\varphi,z,t)\right]-\alpha_{1T}\alpha_{1C}\right\}d\,z\,d\,\varphi\,r\,d\,r\,d\,t-\frac{V_0}{\pi\Theta R^2L}\times$$

$$\times\int_0^{\Theta}(\Theta-t)\int_0^{R}r\int_0^{2\pi}\left[(\alpha_{2C}+C_0)\cdot T_1(r,\varphi,L,t)-\alpha_{1T}\alpha_{1C}\right]d\,\varphi\,d\,r\,d\,t\left\{1+\frac{1}{\pi\Theta RL}\int_0^{\Theta}\int_0^{2\pi}\int_{-L}^{L}v_r(R,\varphi,z,t)\times\right.$$

$$\times\left[\alpha_{2C}+C_1(R,\varphi,z,t)\right]d\,z\,d\,\varphi(\Theta-t)\,d\,t-\frac{1}{\pi\Theta R^2L}\int_0^{\Theta}(\Theta-t)\int_0^{R}r\int_0^{2\pi}\int_{-L}^{L}\left[\alpha_{2C}+C_1(r,\varphi,z,t)\right]\times$$

$$\left.\times\,v_r(r,\varphi,z,\tau)\,d\,z\,d\,\varphi\,d\,r\,d\,t+2V_0(\alpha_{2C}+C_0)\frac{\Theta}{L}\right\}^{-1},$$

$$\alpha_{2C}=\frac{1}{\pi\Theta R^2L}\int_0^{\Theta}(\Theta-t)\int_0^{R}r\int_0^{2\pi}D\left[\left.\frac{\partial C_1(r,\varphi,z,\tau)}{\partial z}\right|_{z=L}-\left.\frac{\partial C_1(r,\varphi,z,\tau)}{\partial z}\right|_{z=-L}\right]d\,\varphi\,d\,r\,d\,t-$$

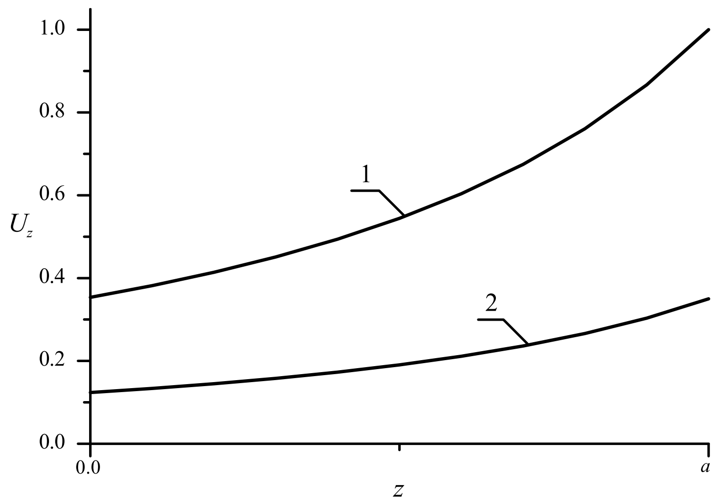
$$-\frac{1}{\pi\Theta R^2L}\int_0^{\Theta}(\Theta-t)\int_0^{2\pi}\int_{-L}^{L}\left\{r\left[\alpha_{2C}-\alpha_{1C}+C_1(R,\varphi,z,\tau)\right]\cdot v_r(R,\varphi,z,\tau)\right\}d\,z\,d\,\varphi\,d\,t-$$

$$-\frac{V_0}{\pi\Theta R^2L}\int_0^{\Theta}(\Theta-t)\int_0^{R}r\int_0^{2\pi}(\alpha_{2C}-\alpha_{1C}+C_0)\,d\,z\,d\,\varphi\,d\,r\,d\,t.$$

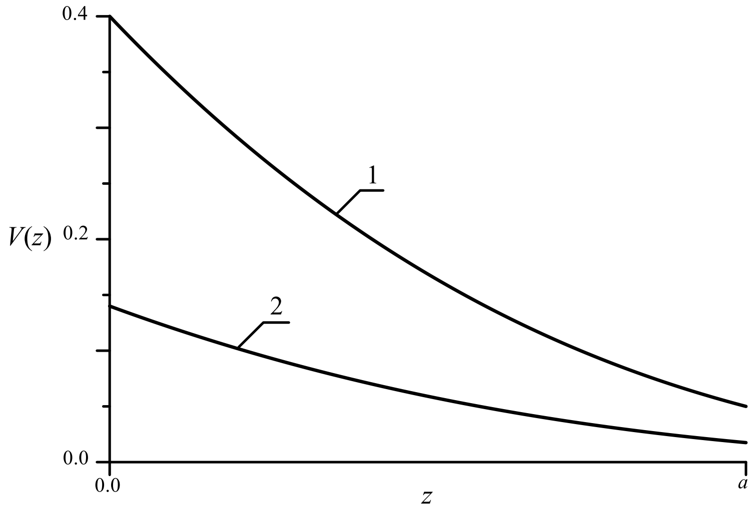
In this paper, we calculate the concentration of radiation defects, components of the displacement vector, and the speed of flow of the mixture of gases-reagents, as well as the growth temperature, using the second-order approximation framework and averaging of function corrections. The approximation is usually good enough to support qualitative analysis and to obtain some quantitative results. All obtained results have been checked against numerical simulation results.

3 | Discussion

In this section, we analyze the dynamics of mass and heat transfer during the growth of films in reactors for epitaxy from the gas phase to determine conditions that improve the properties of epitaxial layers. The main aim of this paper is to analyze the influence of substrate radiation processing on the mismatch-induced stress in the grown heterostructure. *Fig. 2* shows distributions of concentrations in the considered heterostructure (a is the thickness of the epitaxial layer after completion of growth) and dependences of the component of the displacement vector u_z on the coordinate z.



a.



b.

Fig. 2. Normalized distributions in irradiated epitaxial layers: a. displacement component (u_z) along the (z)-coordinate, b. vacancy concentration along the (z)-coordinate for unstressed and stressed layers.

Analysis of the considered processes shows that preliminary radiation processing of the substrate allows decreasing mismatch-induced stress in the considered heterostructure during growth at high temperature. However, one can find a smoother interface between the layers of the heterostructure.

4 | Conclusion

In this paper, we introduce an approach to reduce mismatch-induced stress in a heterostructure by radiation processing of the substrate before the growth of epitaxial layers from the gas phase. In this situation, one can observe a decrease in mismatch-induced stress in the heterostructure. Also, one can observe an acceleration of recombination and the diffusion of radiation defects in the damaged area during growth at high temperature. At the same time, the interface between the layers of the heterostructure became smoother. We also consider an analytical approach for analyzing mass and heat transfer. The approach allows simultaneous consideration of spatial and temporal variations in the parameters of mass transfer. At the same time, the approach allows for the consideration of the nonlinearity of the processes under consideration.

Authors' Contributions

All aspects of the research and manuscript preparation were carried out by the author. The author has read and approved the final version of the manuscript.

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Data Availability

All data supporting the reported findings in this research paper are provided within the manuscript.

Conflict of Interest

The author declares that they have no conflicts of interest.

Consent for Publication

The author confirms consent for the publication of this work.

Ethics Approval and Consent to Participate

This article does not contain any studies with human participants performed by the author.

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