

Fundamental Theorems on Regular Almost-Periodic Functions

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Abstract

The article covers important mathematical properties of uniformly almost-periodic functions and provides proofs of the corresponding theorems. It is shown that a uniformly almost-periodic function is bounded on the entire real axis and, as a consequence, the square of such a function also possesses the uniformly almost-periodic property. It is also proved that these functions are uniformly continuous on the entire real axis. Furthermore, theorems are established demonstrating that the sum and product of uniformly almost-periodic functions retain the same property. The limit of a uniformly convergent sequence of such functions is also shown to be a uniformly almost-periodic function. If the derivative of an almost-periodic function is uniformly continuous on the entire real axis, then its derivative is also uniformly almost-periodic. Finally, it is proved that if an indefinite integral of a uniformly almost-periodic function is bounded, then the integral itself is a uniformly almost-periodic function.

Keywords: Uniformly almost-periodic functions, Boundedness, Uniform continuity, Sums and products, Uniformly convergent sequences, Derivatives, Indefinite integrals.

1|Introduction

Almost-periodic functions constitute an important class of functions that generalize the classical concept of periodicity [1, 2]. The theory was created by H. Bohr in the 1920s [3, 4, 5] and was soon developed by S. Bochner

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[6], A. S. Besicovitch [7], J. Favard [8], J. von Neumann [9] and W. Maak [10]. Systematic expositions are given in the monographs [11, 12, 13]. The theory has become an important area of analysis with applications to differential equations [14, 15, 13, 16, 17], harmonic analysis [18, 19, 20, 21], dynamical systems [13, 15] and functional analysis [22, 23, 24]. Unlike periodic functions, almost-periodic functions need not possess a fixed period. Nevertheless, their values recur with arbitrary accuracy over relatively dense sets of translations. A set $E \subset \mathbb{R}$ is called relatively dense if there exists $\ell > 0$ such that every interval of length ℓ contains at least one point of E .

A continuous function $f : \mathbb{R} \rightarrow \mathbb{C}$ is called uniformly almost-periodic if, for every $\varepsilon > 0$, the set of real numbers τ , called ε -almost-periods, such that

$$|f(x + \tau) - f(x)| < \varepsilon$$

for all $x \in \mathbb{R}$ is relatively dense. The corresponding number $\ell = \ell(\varepsilon)$ is called an inclusion length. This property is a natural generalization of periodicity and gives rise to a rich collection of structural properties [1, 25].

Since Bohr's work, several wider classes have been introduced, namely the Stepanov, Weyl and Besicovitch classes [26, 27, 7, 28], whose mutual relations are analysed in [29], and the almost-automorphic functions of Bochner [30]. Functions with values in Banach spaces are treated in [23, 24, 31, 32], and a number of recent books and papers develop almost periodicity of vector-valued, multidimensional and abstract-group type [33, 34, 35]. A recent survey with more than two hundred references is [36]. The equivalence of the Bohr, Bochner and von Neumann definitions is discussed in [37], and extensions of Bochner's criterion are given in [38]. For the background from real analysis and Fourier series we refer to [25, 39, 40]. The purpose of this paper is to give a self-contained treatment of several fundamental properties of uniformly almost-periodic functions. First, we prove that every uniformly almost-periodic function is bounded on the entire real axis. As an immediate consequence, we show that the square of a uniformly almost-periodic function is also uniformly almost-periodic. We then establish the uniform continuity of uniformly almost-periodic functions on $\mathbb{R}$. Further, we consider algebraic operations on uniformly almost-periodic functions. In particular, it is proved that the sum and product of two uniformly almost-periodic functions preserve the almost-periodic property. We also investigate the behavior of uniformly convergent sequences of uniformly almost-periodic functions and show that their uniform limit is again uniformly almost-periodic [1, 11]. Another aspect considered in this paper is differentiation. We prove that if an almost-periodic function has a derivative which is uniformly continuous on the whole real axis, then the derivative is itself uniformly almost-periodic. Finally, we study indefinite integrals of uniformly almost-periodic functions. In particular, we prove that if an indefinite integral of a uniformly almost-periodic function is bounded on the real axis, then this integral is also uniformly almost-periodic. This is a classical result going back to Bohr [3, 4]; see also [39, 22]. These results describe several basic stability properties of the class of uniformly almost-periodic functions and provide useful tools for further investigations involving differentiation, integration, uniform convergence and Fourier analysis [40, 21].

2|Fundamental Properties of Uniformly Almost-Periodic Functions

Theorem 1. A uniformly almost-periodic function is bounded on the entire real axis.

Proof: Let $f(x)$ be a uniformly almost-periodic function and let $\varepsilon = 1$. Since the set of 1-almost-periods is relatively dense, there is a number $l = l(1)$ such that every interval of length l contains a 1-almost-period. Denote by M_0 the maximum of the continuous function $|f(x)|$ on the closed interval $[0, l]$. [2, 1, 11]

Let x_0 be an arbitrary point of the real axis. Choose an almost-period $\tau = \tau(1)$ in the interval $(-x_0, -x_0 + l)$. Then $0 < x_0 + \tau < l$. Consequently,

$$\begin{aligned} |f(x_0)| &= |f(x_0) - f(x_0 + \tau) + f(x_0 + \tau)| \\ &\leq |f(x_0) - f(x_0 + \tau)| + |f(x_0 + \tau)| \\ &< 1 + M_0 = M. \end{aligned}$$

Thus, $f(x)$ is bounded on the entire real axis. □

Therefore, for every $\varepsilon/(2M) > 0$, an $\varepsilon/(2M)$ -almost-period of $f(x)$ is an ε -almost-period of $f^2(x)$. Hence $f^2(x)$ is uniformly almost-periodic.

Theorem 2. A uniformly almost-periodic function is uniformly continuous on the entire real axis.

Proof: Let $\varepsilon > 0$ be arbitrary and let $l = l(\varepsilon/3)$ be an inclusion length for the $\varepsilon/3$ -almost-periods. The function $f(x)$ is uniformly continuous on the closed interval $[-1, 1 + l]$. Therefore there exists a number $\delta = \delta(\varepsilon/3)$ with $0 < \delta \leq 1$ such that, for arbitrary $y_1, y_2 \in [-1, 1 + l]$, the condition $|y_2 - y_1| < \delta$ implies $|f(y_2) - f(y_1)| < \varepsilon/3$. [2, 1, 11]

Now let x_1 and x_2 be arbitrary real numbers with $|x_2 - x_1| < \delta$. Let τ be an $\varepsilon/3$ -almost-period of $f(x)$ lying in the interval $(-x_1, -x_1 + l)$. Then $0 < x_1 + \tau < l$, and since $|x_2 - x_1| < \delta \leq 1$, the point $x_2 + \tau$ also belongs to $[-1, 1 + l]$. Hence

$$\begin{aligned} |f(x_2) - f(x_1)| &\leq |f(x_2) - f(x_2 + \tau)| + |f(x_2 + \tau) - f(x_1 + \tau)| \\ &\quad + |f(x_1 + \tau) - f(x_1)| \\ &< \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon. \end{aligned}$$

Since ε is arbitrary, $f(x)$ is uniformly continuous on the entire real axis. $\square$

A continuous function $f(x)$ is called a *normal function* if the family $\{f(x + h) : -\infty < h < \infty\}$ is relatively compact with respect to uniform convergence on the entire real axis, that is, every sequence $f(x + h_n)$ has a subsequence which converges uniformly on R .

Theorem 3 (Bochner's criterion). A continuous function $f(x)$ is uniformly almost-periodic if and only if it is a normal function.

This theorem was proved by Bochner [6]; its proof can be found in [1, 11], and generalizations in [36, 33]. We shall use it without repeating the proof.

Theorem 4. The sum and product of uniformly almost-periodic functions are also uniformly almost-periodic.

Proof: Let $f(x)$ and $g(x)$ be uniformly almost-periodic functions. By Theorem 3 the families $\{f(x + h)\}$ and $\{g(x + h)\}$ are relatively compact, and by Theorem 1 both functions are bounded, say $|f|, |g| \leq M$. Consider an arbitrary sequence h_n . First choose a subsequence h'_n such that $f(x + h'_n)$ converges uniformly to some φ . Then, from h'_n , choose a further subsequence h''_n such that $g(x + h''_n)$ converges uniformly to some ψ . Consequently $f(x + h''_n) + g(x + h''_n)$ converges uniformly to $\varphi + \psi$. Moreover,

$$|fg(x + h''_n) - \varphi\psi| \leq M|g(x + h''_n) - \psi| + |\psi||f(x + h''_n) - \varphi| \rightarrow 0$$

uniformly on R , because $|\psi| \leq M$. Therefore the families $\{f(x + h) + g(x + h)\}$ and $\{f(x + h)g(x + h)\}$ are relatively compact, and by Theorem 3 the functions $f(x) + g(x)$ and $f(x)g(x)$ are uniformly almost-periodic. $\square$

Theorem 5. Let $f_1(x), f_2(x), \dots$ be a sequence of uniformly almost-periodic functions which converges uniformly on R , and let $f(x) = \lim_{n \rightarrow \infty} f_n(x)$. Then $f(x)$ is uniformly almost-periodic.

Proof: The limit f is continuous, being a uniform limit of continuous functions. For an arbitrary positive number ε there exists an integer $N = N(\varepsilon)$ such that

$$\sup_x |f(x) - f_N(x)| < \frac{\varepsilon}{3}.$$

Let τ be an $\varepsilon/3$ -almost-period of the uniformly almost-periodic function $f_N(x)$. Then

$$\begin{aligned} |f(x + \tau) - f(x)| &\leq |f(x + \tau) - f_N(x + \tau)| + |f_N(x + \tau) - f_N(x)| \\ &\quad + |f_N(x) - f(x)| < \varepsilon. \end{aligned}$$

Thus every $\varepsilon/3$ -almost-period of f_N is an ε -almost-period of f , and $f(x)$ is uniformly almost-periodic. $\square$

Theorem 6. Let $f(x)$ be a uniformly almost-periodic function which is differentiable on R , and suppose that $f'(x)$ is uniformly continuous on R . Then $f'(x)$ is uniformly almost-periodic.

Proof: For $h \neq 0$ put

$$g_h(x) = \frac{f(x+h) - f(x)}{h}.$$

Every ε -almost-period of f is also an ε -almost-period of the translate $f(x+h)$, so $f(x+h)$ is uniformly almost-periodic. By Theorem 4 (and since multiplication by a constant preserves almost periodicity), $g_h(x)$ is uniformly almost-periodic for each $h \neq 0$. The derivative f' is continuous, hence

$$g_h(x) - f'(x) = \frac{1}{h} \int_0^h (f'(x+s) - f'(x)) ds.$$

Let $\omega(\eta) = \sup\{|f'(u) - f'(v)| : |u - v| \leq \eta\}$ be the modulus of continuity of f' ; by assumption $\omega(\eta) \rightarrow 0$ as $\eta \rightarrow 0$. Then $\sup_x |g_h(x) - f'(x)| \leq \omega(|h|) \rightarrow 0$ as $h \rightarrow 0$. Thus $g_{1/n} \rightarrow f'$ uniformly on R , and Theorem 5 shows that f' is uniformly almost-periodic. $\square$

3|Indefinite Integrals of Uniformly Almost-Periodic Functions

For a periodic function $f(x)$, a necessary and sufficient condition for its indefinite integral to be periodic is that the constant term (the mean value) in the Fourier series of $f(x)$ be equal to zero [38, 37].

A similar question arises for uniformly almost-periodic functions. Such functions also possess Fourier series, and the mean value $\lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(x) dx$ exists [2, 1, 18]. However, in general, the vanishing of the mean value is not sufficient to guarantee that the indefinite integral is almost-periodic.

On the other hand, if the indefinite integral F of a uniformly almost-periodic function is itself uniformly almost-periodic, then F must be bounded on the entire real axis by Theorem 1. The following theorem shows that this necessary condition is also sufficient.

Theorem 7. If the indefinite integral of a uniformly almost-periodic function is bounded, then it is also a uniformly almost-periodic function.

Proof: By assumption,

$$F(x) = \int_0^x f(t) dt + C$$

is a bounded function. Denote its least upper bound and greatest lower bound by G and g , respectively. If $G = g$, then F is constant and there is nothing to prove, so let $G > g$. It suffices to consider ε with $0 < \varepsilon < 3(G - g)$, because an ε' -almost-period with $\varepsilon' < \varepsilon$ is also an ε -almost-period. We shall show that there exists a positive number $\varepsilon_1 = \varepsilon_1[\varepsilon, f]$ such that every ε_1 -almost-period of $f(x)$ is an ε -almost-period of $F(x)$.

Choose two fixed numbers x_1 and x_2 satisfying

$$F(x_1) < g + \frac{\varepsilon}{6}, \quad F(x_2) > G - \frac{\varepsilon}{6}.$$

Then $F(x_2) - F(x_1) > G - g - \varepsilon/3 > 0$, so $x_1 \neq x_2$. Let $d = |x_1 - x_2| > 0$ and $m = \min(x_1, x_2)$. Let $l_0 = l(\frac{\varepsilon}{6d})$ be an inclusion length for the $\frac{\varepsilon}{6d}$ -almost-periods of f , and put $L_0 = l_0 + d$.

Step 1. We show that every interval $(a, a + L_0)$ contains points z_1 and z_2 such that

$$F(z_1) < g + \frac{\varepsilon}{2}, \quad F(z_2) > G - \frac{\varepsilon}{2}. \quad (1)$$

Choose an $\frac{\varepsilon}{6d}$ -almost-period τ such that $m + \tau \in (a, a + l_0)$. Then both numbers $z_1 = x_1 + \tau$ and $z_2 = x_2 + \tau$ lie in $[m + \tau, m + \tau + d] \subset (a, a + L_0)$, and

By the meaning of G and g , the inequality $F(z_2) - F(z_1) > G - g - \varepsilon/2$ can hold only if both inequalities (1) are satisfied.

Step 2. Put $\varepsilon_1 = \varepsilon/(2L_0)$ and let τ be an ε_1 -almost-period of $f(x)$. We show separately that for every real x

$$F(x + \tau) - F(x) > -\varepsilon, \quad (2)$$

$$F(x + \tau) - F(x) < \varepsilon. \quad (3)$$

To prove (2), choose by Step 1 a point $z_1 \in (x, x + L_0)$ with $F(z_1) < g + \varepsilon/2$. Then

$$\begin{aligned} F(x + \tau) - F(x) &= F(z_1 + \tau) - F(z_1) + \int_x^{z_1} \{f(y) - f(y + \tau)\} dy \\ &> g - \left(g + \frac{\varepsilon}{2}\right) - L_0 \varepsilon_1 = -\frac{\varepsilon}{2} - \frac{\varepsilon}{2} = -\varepsilon. \end{aligned}$$

To prove (3), choose a point $z_2 \in (x, x + L_0)$ with $F(z_2) > G - \varepsilon/2$. Then

$$\begin{aligned} F(x + \tau) - F(x) &= F(z_2 + \tau) - F(z_2) + \int_x^{z_2} \{f(y) - f(y + \tau)\} dy \\ &< G - \left(G - \frac{\varepsilon}{2}\right) + L_0 \varepsilon_1 = \varepsilon. \end{aligned}$$

Hence $|F(x + \tau) - F(x)| < \varepsilon$ for all $x \in \mathbb{R}$. Since the ε_1 -almost-periods of f form a relatively dense set, so do the ε -almost-periods of the continuous function F , and F is uniformly almost-periodic. $\square$

Corollary 1 (Bohr's criterion). The indefinite integral of a uniformly almost-periodic function f is uniformly almost-periodic if and only if it is bounded on $\mathbb{R}$.

Necessity follows from Theorem 1, sufficiency from Theorem 7.

Remarks and Further Directions

Remark. Theorems 4, 5 and 6, together with the fact that translates and complex conjugates of uniformly almost-periodic functions are again uniformly almost-periodic, show that these functions form a translation-invariant closed subalgebra of the Banach algebra of bounded continuous functions on $\mathbb{R}$ with the supremum norm. By Bohr's approximation theorem it coincides with the uniform closure of the set of trigonometric polynomials [2, 1, 18].

Remark. The properties established above remain valid, with essentially the same proofs, for functions with values in a Banach space, except for Theorem 7: for infinite-dimensional spaces, boundedness of the integral is in general not sufficient and additional assumptions on the space (for instance, the absence of subspaces isomorphic to c_0) are required [12, 23, 24].

Remark. In applications, Theorems 6 and 7 are used to prove the existence of almost-periodic solutions of ordinary and abstract differential equations, where bounded solutions on $\mathbb{R}$ are shown to be almost-periodic [14, 13, 16, 17, 31]. Extending the results of this paper to the Stepanov, Weyl and Besicovitch classes [?, 25, 7, 27] and to almost-automorphic functions [28, 30] is a natural direction for further research.

4|Conclusion

In this article, we have presented self-contained proofs of several fundamental properties of uniformly almost-periodic functions: boundedness, uniform continuity, closedness under sums, products, squares and uniform limits, almost periodicity of uniformly continuous derivatives, and almost periodicity of bounded indefinite integrals. These properties show that the class of uniformly almost-periodic functions is stable under the basic operations of analysis, which makes it a convenient framework for the study of differential and integral equations and of harmonic analysis on the real line. As directions for further research we indicated generalizations to Banach-space-valued functions and to the wider classes of Stepanov, Weyl, Besicovitch and almost-automorphic functions.

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Author Contribution

F. Mammadov: methodology, software, and editing. S. Agaliyeva: conceptualization. J. Aliyev: writing and editing. All authors have read and agreed to the published version of the manuscript.

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