Karshi Multidisciplinary International Scientific Journal

www.kmisj.reapress.com

K. Multidiscip. Int. Sci. J. Vol. 3, No. 3 (2026) 233–240.

Paper Type: Original Article

Movement of An Elastic Liquid In An Elastic Semi-Infinite Layer With A Poorly Permeable Roof

Ilgar S. Safarli[1,*], **Vusala N. Nasibzadeh**[1], **Ilaha I. Safarli**[1]

[1]Sumgayit State University, Sumgayit, Azerbaijan; Ilqar.Safarli@sdu.edu.az; vusala.nasibzada@sdu.edu.az; ilahe.seferli@sdu.edu.az.

Citation:

Received: 22 November 2025 Revised: 04 February 2026 Accepted: 19 April 2026	Safarli, I. S., Nasibzadeh, V. N., & Safarli, I. I. (2026). Movement of an Elastic liquid in an Elastic semi-infinite layer with a poorly permeable roof. *Karshi Multidisciplinary International Scientific Journal*, 3(3), 233-240.

Abstract

In this article, we consider modeling and pricing a combination of two options (Exchange option and Lookback option) that we call the LOEX option. It is a type of exotic option, or clearer, path-dependent option because its price depends on the maximum and minimum prices of the assets to be considered. In reality, we introduce a conditional claim on two assets: the holder can change the first asset with the highest price to the second with the lowest price or cancel the transaction at the strike time. In this paper, after describing and modeling this option, we use numerical methods to estimate the pricing using the MATLAB software and present the results.

Keywords: Mathematics, Mechanics, Analytical expressions, Laplace transforms, Fluid dynamics.

1|Introduction

When drafting rational development design systems for oil, gas, and gas-condensate fields, it is of great importance to account for the heterogeneity of productive formations, as well as potential cross-flows of liquid from one formation to another through poorly permeable interlayers between them [4, 5].

Corresponding Author: Ilqar.Safarli@sdu.edu.az

https://doi.org/10.22105/kmisj.vi.146

Licensee System Analytics. This article is an open-access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0).

To perform calculations, it is necessary to have analytical expressions obtained by mathematical methods for the distribution of the pressure function in the corresponding formations. The derivation of such analytical expressions is dedicated to works [1,2]. Not that in [9, 10] some related problems were considered.

2|Mathematical Formulation of the Problem

It is assumed that in a semi-infinite formation, there is a fault line on one side. Furthermore, in the formation there is a semi-circular zone whose diameter coincides with the fault line. Inside the semi-circular zone (Zone I), the permeability and piezo conductivity coefficients have different values than in the rest of the formation (Zone II). The unsteady hydrodynamic processes in such fractured-porous and heterogeneous formations can be mathematically modeled using non-stationary filtration theory [4, 5].

At the initial moment of time, the pressure throughout the formation is considered constant and equal to P_0. In the overlying formation, the pressure is also considered constant and equal to P_0. A well operating with a given variable flow rate $Q_0(t)$ is located in Zone I.

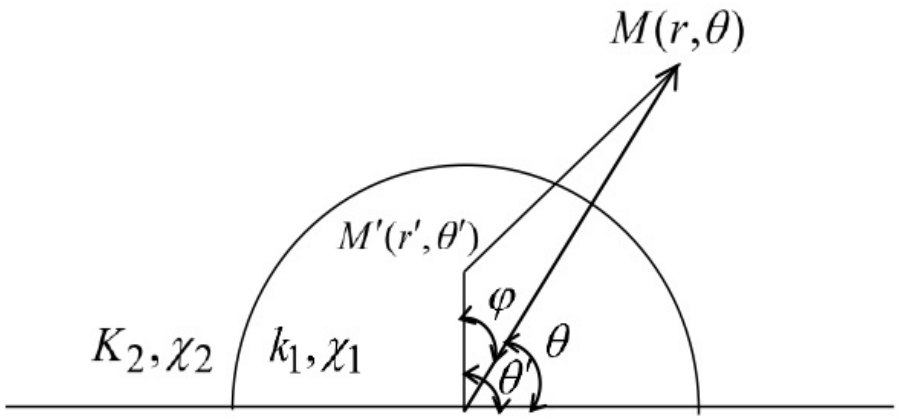


Fig.1

The problem under consideration can be mathematically formulated as follows:

$$\Delta P_i - \alpha_i^2 (P_i - P_0) = \frac{1}{\chi_i} \frac{\partial P_i}{\partial t}, \tag{1}$$

under the following additional conditions

$$P_1(r,\theta,t) = P_2(r,\theta,t) = P_0 \quad \text{for} \quad t = 0 \tag{2}$$

$$\left(R \frac{\partial P_i}{\partial R} \right)_{R \to 0} = \frac{MQ^*(t)}{2\pi b k_i} = Q(t) \tag{3}$$

$$P_1(r,\theta,t) = P_2(r,\theta,t) \quad \frac{\partial P_1}{\partial r} = \nu \frac{\partial P_2}{\partial t} \quad \text{for} \quad r = a \tag{4}$$

$$P_2(r,\theta,t) = P_0 \quad \text{for} \quad r = \infty \tag{5}$$

where

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{2}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}; \quad i = 1,2; \quad \alpha_i^2 = \frac{K_{\Gamma_i}}{k_i b b_\Gamma};$$

$$0 \le \theta \le \pi; \quad t > 0; \quad r \in \begin{cases} [0,a], & i = 1 \\ [a,\infty), & i = 2 \end{cases}$$

$$R_\pm^2 = r^2 + r'^2 - 2rr' \cos(\theta \pm \theta').$$

$Q^*(t)$ is the well flow rate, index $i = 1$ refers to Zone I, and $i = 2$ to Zone II, P_i is the thickness-averaged value of the pressure function in the corresponding zone, (r, θ) are cylindrical coordinates, t is time, (r', θ') are the coordinates of the formation point where the well is located, k_i and χ_i are the permeability and piezo conductivity coefficients, b and b_G are the thicknesses of the formation and the interlayer, $K_{\Gamma i}$ is the permeability coefficient of the interlayer above the corresponding zone, which by the condition of the problem $K_\Gamma << k_i$.

To solve the posed system of partial differential equations (1)–(5), we apply the Laplace integral transform method [6, 10]:

$$\bar{P}_i(r, \theta, \lambda) = \int_0^\infty R^{-\lambda t} P\,(r, \varphi, t)\, dt \tag{6}$$

where λ is the transform parameter. $\bar{P}_i$ is the image of the function P_i, after which, taking into account the initial condition (2), we obtain

$$\Delta u_i - q_i^2 u_i = 0 \tag{7}$$

where

$$q_i^2 = \alpha_i^2 + \frac{\lambda}{\chi_i}, \quad u_i = \frac{1}{\lambda} P_0 - \bar{P}_i$$

Similarly to works [1, 2], the solution to the problem for the considered case in Laplace transform images (7) can be presented in the following form [8] :

$$u_1 = \bar{Q}(\lambda) \left\{ K_0(q_1 R_-) + K_0(q_1 R_+) + \sum_{n=0}^{\infty} \varepsilon_n I_n\,(q_1 r) \left[a_n^{(1)} \cos n\varphi + a_n^{(2)} \sin n\varphi \right] \right\},$$

$$u_2 = \bar{Q}(\lambda) \sum_{n=0}^{\infty} \varepsilon_n K_n\,(q_2 r) \left[b_n^{(1)} \cos n\varphi + b_n^{(2)} \sin n\varphi \right] \tag{8}$$

The unknown coefficients in (8) are determined from the corresponding boundary conditions (having preliminarily applied the Laplace transform to them) in the form

$$a_n^{(1)} = 2\varepsilon_n I_n\,(q_1 r')\, \Phi_n^{(1)}\,(a, \lambda) \cos n\theta',$$

$$b_n^{(1)} = -\tfrac{2\varepsilon_n}{q_1 a} I_n(q_1 r') \Phi_n^{(2)}\,(a, \lambda) \cos n\theta',$$

$$a_n^{(2)} = b_n^{(2)} = 0$$

where

$$\Phi_n^{(1)}(a, \lambda) = K_n'(q_1 r) K_n(q_2 a) - \sigma K_n(q_1 r) K_n'(q_2 a),$$

$$\Phi_n^{(2)}(a, \lambda) = \left[\sigma I_n(q_1 a) K_n'(q_2 a) - I_n'(q_1 a) K_n(q_2 a)\right]^{-1},\ \sigma = \nu \tfrac{q_2}{q_1}, \nu = \tfrac{k_2}{k_1}$$

Taking into account the found values of the coefficients in (8) for pressure drops, we obtain:

$$\begin{aligned} u_1 &= \bar{Q}(\lambda) \bar{B}_1(r, \theta, \lambda), \\ u_2 &= \bar{Q}(\lambda) \bar{B}_2(r, \theta, \lambda) \end{aligned} \tag{9}$$

where

$$\bar{B}_1(r, \theta, \lambda) = 2 \sum_{n=0}^{\infty} \varepsilon_n \left[\Phi_n^{(1)}(a, \lambda) I_n(q_1 r') K_n(q_1 r) + I_n(q_1 r') K_n'(q_1 r) \right] \cos n\theta \cos n\theta',$$

$$\bar{B}_2(r, \theta, \lambda) = 2 \sum_{n=0}^{\infty} \tfrac{\varepsilon_n}{q_1 a} \Phi_n^{(2)}(a, \lambda) I_n(q_1 r') K_n(q_2 r) \cos n\theta \cos n\theta'$$

Taking into account the found values of the coefficients in (8) by the Laplace transform formula

$$P_i(r, \theta, t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{\lambda t} P(r, \theta, \lambda) d\lambda (9/)$$

Passing to the originals, we obtain (10), where we use the well-known expansion of the product of cylindrical functions [3]:

$$P_0 - P_1 = \frac{8}{\pi^2} \sum_{n=0}^{\infty} \varepsilon_n \cos n\theta \cos n\theta' \int_0^{\infty} \frac{\nu J_n(u\bar{r}) J_n(u\bar{r}') f(u,t)}{u(M_n^2+N_n^2)} du,$$
$$P_0 - P_2 = \frac{4}{\pi} \sum_{n=0}^{\infty} \varepsilon_n \cos n\theta \cos n\theta' \int_0^{\infty} \frac{M_n Y_n(\rho\bar{r}) - N_n J_n(\rho\bar{r})}{M_n^2+N_n^2} J_n(u\bar{r}') f(u,t) du \tag{10}$$

where we use the well-known expansion of the product of cylindrical functions [3]

$$K_0(qr) = \sum_{n=0}^{\infty} \varepsilon_n \cos n\varphi I_n(q\bar{r}) K_n(qr')$$

where

$$\varepsilon_n = \begin{cases} 1, & n = 0 \\ 2, & n \geq 1 \end{cases}$$

if $r > r'$, and if $r > r'$, then r and r' must be interchanged in the formula.

$$u(\rho\bar{r}) = M_n Y_n(\rho\bar{r}) - N_n J_n(\rho\bar{r}),$$
$$M_n = \sigma_0 J_n(u) J_n'(p) - J_n'(u) J_n(p), \quad N_n = \sigma_0 J_n(u) Y_n'(p) - J_n'(u) Y_n(p),$$
$$\rho = \frac{1}{\sqrt{\chi_2}} \sqrt{\chi_2 \alpha_2^2 - \alpha_1^2 \chi_1 + \chi_1 u^2}, \quad \bar{r} = \frac{r}{a}, \quad \bar{r}' = \frac{r'}{a}, \quad \sigma_0 = \nu \frac{\rho}{u},$$
$$f(u,t) = \int_0^t \exp\left[-\chi_1 (t-\tau)\eta\right] Q(\tau) d\tau, \qquad \eta = \frac{u^2}{a^2} + \alpha_1^2,$$

J_n, Y_n are cylindrical functions of a real argument of the first and second kind of the n-th order, respectively.

Let us consider special cases.

If in the first formula (8) as $k_2 \to 0 \quad (q_2 \to \infty)$ we pass to the limit, then the resulting formula will correspond to the image of the pressure drop function in a closed semi-circular formation with radius a and with a well located at point (r', θ') (Fig. 1), or to the case of a closed circular formation with two wells operating with equal flow rates and located symmetrically relative to the line $\theta = 0$ at points (r', θ') and $(r', -\theta')$. This formula has the form:

$$u_1 = \bar{Q}(\lambda) \left[K_0(q_1 R_-) + K_0(q_1 R_+) - \right.$$
$$\left. -2 \sum_{n=0}^{\infty} \frac{\varepsilon_n I_n(qr') I_n(q_1 r) K_n'(qa)}{I_n'(qa)} \cos n\theta \cos n\theta' \right]. \tag{11}$$

Passing to the original, we obtain:

$$P_0 - P_1 = \frac{4\chi_1}{a^2} \sum_{n=0}^{\infty} \varepsilon_n \cos n\theta \cos n\theta' \sum_{m=1}^{\infty} \alpha_{n,m}^2 \frac{J_n\left(\alpha_{n,m}\bar{r}'\right) J_n\left(\alpha_{n,m}\bar{r}\right)}{\left(\alpha_{n,m}^2 - n^2\right) J_n^2(\alpha_{n,m})} E(\alpha_{n,m} t) \tag{12}$$

where

$$E(\alpha_{n,m} t) = \int_0^t \exp\left[-\chi_1 \left(\frac{\alpha_{n,m}^2}{a^2} + \alpha_1^2 \right) \tau \right] Q(t-\tau) d\tau$$

$\alpha_{n,m}$ are the positive roots of the equation

$$J_n'(\alpha) = 0$$

Now in (8), passing to the limit as $k_2 \to \infty \quad (q_2 \to 0)$, we will have:

$$u_1 = \bar{Q}(\lambda) \left[K_0(q_1 R_-) + K_0(q_1 R_+) - \right.$$
$$\left. - 2 \sum_{n=0}^{\infty} \varepsilon_n \frac{I_n(q_1 r') I_n(q_1 r) K_n'(q_1 a)}{I_n'(q_1 a)} \cos n\theta \cos n\theta' \right]. \tag{13}$$

Formula (13) is the image of the pressure drop function in a semi-circular formation having a fault line on one side (along the diameter) and a feed contour on the other side (along the semi-circle), with a well located at

point (r', θ'), or corresponds to the case where in an open circular formation there are two wells with equal variable flow rates located symmetrically relative to the fault line at points (r', θ') and $(r', -\theta')$.

In [7, 8], passing to the original, we obtain:

$$P_0 - P_1 = \frac{4\chi_1}{a^2} \sum_{n=0}^{\infty} \varepsilon_n \cos n\theta \cos n\theta' \sum_{m=1}^{\infty} \frac{J_n\left(\alpha_{n,m}\bar{r'}\right) J_n\left(\alpha'_{n,m}\bar{r}\right)}{J_n^2(a_{n,m})} E(\alpha_{n,m}t) \tag{14}$$

where $\alpha_{n,m}$ are the positive roots of the equation

$$J_n(\alpha) = 0$$

If the well operates with a constant flow rate, i.e., if $Q(t) = const$, then $f(u,t)$ and $E(\alpha_{n,m}t)$ will have the form

$$f(u,t) = \frac{Qa^2}{\chi_1\left(u^2+\alpha_1^2a^2\right)} \left\{1 - \exp\left[-\frac{\chi_1 t}{a^2}(u^2 + \alpha_1^2a^2)\right]\right\},$$
$$E(\alpha_{n,m}t) = \frac{Qa^2}{\chi_1\left(\alpha_{n,m}^2+\alpha_1^2a^2\right)} \left\{1 - \exp\left[-\frac{\chi_1 t}{a^2}(\alpha_{n,m}^2 + \alpha_1^2a^2)\right]\right\}$$

Taking into account the constancy of the well flow rate from (13), we will have:

$$P_0 - P_1 = Q\left\{P_{cm}^{(1)}(r,\theta) - 4\sum_{n=0}^{\infty} \varepsilon_n \cos n\theta \cos n\theta' \times \right.$$
$$\left. \times \sum_{m=1}^{\infty} \frac{\alpha_{n,m}^2 J_n\left(\alpha_{n,m}\bar{r'}\right) J_n\left(\alpha'_{n,m}\bar{r}\right) \exp\left[-\chi_1\left(\frac{\alpha_{n,m}^2}{a^2} + \alpha_1^2\right)t\right]}{\left(\alpha_{n,m}^2 + \alpha_1^2a^2\right)\left(\alpha_{n,m}^2 - n^2\right) J_n^2(a_{n,m})} \right\} \tag{15}$$

where

$$P_{cm}^{(1)}(r,\theta) = K_0(\alpha_1 R_-) + K_0(\alpha_1 R_+) -$$
$$-2\sum_{n=0}^{\infty} \varepsilon_n \cos n\theta \cos n\theta' \frac{I_n\left(\alpha, \bar{r'}\right) I_n\left(\alpha, \bar{r}\right) K'_n(\alpha, a)}{I'_n(\alpha, a)} \tag{16}$$

In exactly the same way, from (14) we will have:

$$P_0 - P_1 = Q\left\{P_{cm}^{(2)}(r,\theta) - 4\sum_{n=0}^{\infty} \varepsilon_n \cos n\theta \cos n\theta' \times \right.$$
$$\left. \times \sum_{m=1}^{\infty} \frac{\alpha_{n,m}^2 J_n\left(\alpha_{n,m}\bar{r'}\right) J_n\left(\alpha_{n,m}\bar{r}\right) \exp\left[-\chi_1\left(\frac{\alpha_{n,m}^2}{a^2} + \alpha_1^2\right)t\right]}{\left(\alpha_{n,m}^2 + \alpha_1^2a^2\right) J_n^2(\alpha_{n,m})} \right\} \tag{17}$$

where

$$P_{cm}^{(2)}(r,\theta) = K_0(\alpha_1 R_-) + K_0(\alpha_1 R_+) -$$
$$-2\sum_{n=0}^{\infty} \varepsilon_n \frac{I_n\left(\alpha, r\right) I_n\left(\alpha, r'\right) K_n(\alpha, a)}{I_n(\alpha, a)} \cos n\theta \cos n\theta' \tag{18}$$

For values of $\alpha a \ll 1$ (due to their smallness), expressions (15) and (18) can be simplified with sufficient accuracy for calculations. Then we will have

$$P_{cm}^{(1)}(r,\theta) = 2\left(\ln\frac{2}{\alpha_1 r} - c\right) + \frac{4}{(\alpha_1 a)^2} - \frac{1}{2}\left[F\left(\frac{r}{r'}, \theta \pm \theta'\right) + F\left(\frac{rr'}{a^2}, \theta \pm \theta'\right)\right], \tag{19}$$

$$P_{cm}^{(2)}(r,\theta) = \frac{1}{2}F\left[a^4\left(\frac{rr'}{a^2}, \theta \pm \theta'\right)\right] - \frac{1}{2}F\left[r^2\left(\frac{r'}{r}, \theta \pm \theta'\right)\right], \tag{20}$$

where

$$F(z, \theta \pm \theta') = \ln\left[1 - 2z\cos\left(\theta + \theta'\right) + z^2\right] \ \left[1 - 2z\cos\left(\theta + \theta'\right) + z^2\right],$$
$$F\left[y(z, \theta \pm \theta')\right] = \ln\left\{y\left[1 - 2z\cos\left(\theta + \theta'\right) + z^2\right]\right\} \ \left\{y\left[1 - 2z\cos\left(\theta - \theta'\right) + z^2\right]\right\}$$

for

$$z=\frac{r}{r'},\frac{rr'}{a^2};\quad y=a^4,r^2$$

$c=0,5772....$ is Euler's constant.

Similarly, formulas (10) can be simplified for large values of time. For this purpose, in the image functions (9) for small values of the parameter λ, which correspond to large values of time in the original, by replacing the cylindrical functions with their expressions for small values of the argument (except for functions with zero indices), we obtain:

$$u_1=\bar{Q}(\lambda)\,2\left\{\left[K_0(q_1r)-K_0(q_1a)+\frac{1}{\nu}K_0(q_2a)\right]-\right.$$
$$\left.-\frac{1}{2}F\left(\frac{r'}{r},\theta\pm\theta'\right)-\frac{1}{2}\frac{1-\nu}{1+\nu}F\left(\frac{rr'}{a^2},\theta\pm\theta'\right)\right\},$$
$$u_2=\bar{Q}(\lambda)\left\{\frac{2}{\nu}K_0(q_2r)-\frac{1}{2}F\left(\frac{r'}{r},\theta\pm\theta'\right)\right\}$$

The originals of these image functions will be:

$$P_0-P_1=\int_0^t\frac{1}{\tau}\left[\exp\left(-\alpha_1^2\chi_1\tau-\frac{r^2}{4\chi_1t}\right)-\exp\left(-\alpha_1^2\chi_1\tau-\frac{a^2}{4\chi_1\tau}\right)+\right.$$
$$\left.+\frac{1}{\nu}\exp\left(-\alpha_2^2\chi_2\tau-\frac{a^2}{4\chi_2t}\right)\right]Q(t-\tau)d\tau-$$
$$-\frac{1}{2}Q(t)\left[F\left(\frac{r'}{r},\theta\pm\theta'\right)+\frac{1-\nu}{1+\nu}F\left(\frac{rr'}{a^2},\theta\pm\theta'\right)\right] \tag{21}$$
$$P_0-P_2=\frac{1}{\nu}\int_0^t\frac{1}{\tau}\exp\left(-\alpha_2^2\chi_2\tau-\frac{r^2}{4\chi_2\tau}\right)Q(t-\tau)d\tau-\frac{1}{2}Q(t)F\left(\frac{r'}{r},\theta\pm\theta'\right)$$

If the well operates with a constant flow rate, i.e., if $Q(t)=Q=const$, then formulas (21) take the form:

$$P_0-P_1=Q\left\{2\left[L\left(\alpha_1,\frac{r^2}{4\chi_1t}\right)-L\left(\alpha_1,\frac{a^2}{4\chi_1t}\right)+\frac{1}{\nu}L\left(\alpha_2,\frac{a^2}{4\chi_2t}\right)\right]-\right.$$
$$\left.-\frac{1}{2}F\left(\frac{r'}{r},\theta\pm\theta'\right)-\frac{1}{2}\frac{1-\nu}{1+\nu}F\left(\frac{rr'}{a^2},\theta\pm\theta'\right)\right\},$$
$$P_0-P_2=Q\left\{\frac{2}{\nu}L\left(\alpha_2,\frac{r^2}{4\chi_2t}\right)-\frac{1}{2}F\left(\frac{r'}{r},\theta\pm\theta'\right)\right\} \tag{22}$$

Passing to the limit $\alpha_1=\alpha_2\to0$ from (22), we obtain:

$$P_0-P_1=Q\left\{Ei\left(-\frac{a^2}{4\chi_1t}\right)-Ei\left(-\frac{r^2}{4\chi_1t}\right)-\frac{1}{\nu}Ei\left(-\frac{a^2}{4\chi_2t}\right)-\right.$$
$$\left.-\frac{1}{2}F\left(\frac{r'}{r},\theta\pm\theta'\right)-\frac{1}{2}\frac{1-\nu}{1+\nu}F\left(\frac{rr'}{a^2},\theta\pm\theta'\right)\right\},$$
$$P_0-P_2=Q\left\{-\frac{1}{\nu}Ei\left(-\frac{r^2}{4\chi_2t}\right)-\frac{1}{2}F\left(\frac{r'}{r},\theta\pm\theta'\right)\right\}. \tag{23}$$

Formulas (23) correspond to the case of impermeability of the roof and bottom of a heterogeneous semi-circular formation with a fault line along the diameter and with a well located in Zone I.

Now, if in (22) as $t\to\infty$ we pass to the limit, we obtain the formulas for steady-state filtration of the considered problem:

$$P_0-P_1=Q\left\{2\left[K_0(\alpha_1r)-K_0(\alpha_1a)+\frac{1}{\nu}K_0(\alpha_2a)\right]-\right.$$

$$-\frac{1}{2}F\left(\frac{r'}{r},\theta\pm\theta'\right)-F\left(\frac{rr'}{a^2},\theta\pm\theta'\right)\Bigg\},$$

$$P_0-P_2=Q\left\{\frac{2}{\nu}K_0\left(\alpha_2 r\right)-\frac{1}{2}F\left(\frac{r'}{r},\theta\pm\theta'\right)\right\}. \tag{24}$$

Due to the smallness of α in formulas (24), they can be further simplified with sufficient accuracy for calculations. Then we obtain:

$$\begin{aligned} P_0-P_1&=Q\left\{2\left[\ln\tfrac{a}{r}+\tfrac{1}{\nu}\left(\ln\tfrac{2}{\alpha_2 a}-c\right)\right]-\tfrac{1}{2}F\left(\tfrac{r'}{r},\theta\pm\theta'\right)-\tfrac{1}{2}\tfrac{1-\nu}{1+\nu}F\left(\tfrac{rr'}{a^2},\theta\pm\theta'\right)\right\},\\ P_0-P_2&=Q\left\{\tfrac{2}{\nu}\left(\ln\tfrac{2}{\alpha_2 a}-c\right)-\tfrac{1}{2}F\left(\tfrac{r'}{r},\theta\pm\theta'\right)\right\}. \end{aligned} \tag{25}$$

Calculation of Bottomhole Pressure Drop

Let us calculate the pressure drop at the well. For this, denoting the bottomhole pressure of the wells by P_c, and taking $r=r'+r_c$ and $\theta=\theta'=\frac{\pi}{4}$ from the first formula (25), we obtain:

$$P_0-P_1=Q\left\{2\left[\ln\frac{a}{r'}+\frac{1}{\nu}\left(\ln\frac{2}{\alpha_2 a}-c\right)\right]-\frac{1}{2}\ln\frac{2r_c^2}{r_1^2}-\right.$$
$$\left.-\frac{1}{2}\frac{1-\nu}{1+\nu}\ln\left(1-\frac{r'^2}{a^2}\right)\left(1+\frac{r'^4}{a^4}\right)\right\}, \tag{26}$$

If we take $r'=\frac{a}{r}$, then we will have:

$$P_0-P_1=Q\left\{\frac{2}{\nu}\left(\ln\frac{r}{\alpha_1 a}-c\right)+\frac{r}{2}\ln 2+\ln\frac{a}{r_c}-\frac{1}{2}\frac{1-\nu}{1+\nu}\ln\frac{153}{256}\right\} \tag{27}$$

As can be seen from the expressions of the formulas obtained above, they are very convenient for conducting hydrodynamic calculations.

3|Conclusion

In this paper, an analytical solution is obtained for the unsteady movement of an elastic liquid in a heterogeneous semi-infinite formation with a poorly permeable roof, taking into account the presence of a semi-circular zone with different filtration properties and a time-variable well flow rate [9].

The main results of the study are summarized as follows:

(1) **Mathematical Solution:** Using the Laplace integral transform and the separation of variables method, analytical expressions were derived for the spatial-temporal pressure distribution across different heterogeneous zones of the formation [9].

(2) **Asymptotic Simplifications:** Analytical solutions for both small and large time values were developed. For large values of time and small parameter values, complex expressions involving cylindrical functions were simplified into highly practical logarithmic forms for engineering applications [9].

(3) **Special Boundary Cases:** The derived general formulas were effectively reduced to closed forms for specific hydrodynamic boundary conditions, including closed semi-circular reservoirs, reservoirs with a constant feed contour, and cases with constant well flow rates [9].

Practical Application: Explicit formulas for calculating the pressure drop at the well bottomhole were derived. These compact analytical relationships allow for direct calculation of pressure dynamics

and filtration parameters without requiring complex numerical integration, making them convenient for designing and optimizing the development of heterogeneous oil and gas fields [9].

References

[1] Guseynov, G. P., & Veliev, M. N. (1967). Dvizhenie uprugoi zhidkosti v uprugom kusochno-odnorodnom plaste [Motion of an Elastic fluid in an Elastic piecewise-homogeneous formation]. *Trudy AzNII Po Dobyche Nefti*, (18), 226–244. (In Russian).

[2] Veliev, M. N., & Jabbarov, I. I. (1972). Vzaimodeistvie skvazhin, raspolozhennykh v Neodnorodnom plaste s slabopronitsaemoi krovlei [Interaction of wells located in a heterogeneous reservoir with a low-permeability cap rock]. Izvestiya Akademii Nauk Azerbaidzhanskoi SSR. *Seriya Fiziko-Tekhnicheskikh i Matematicheskikh Nauk*, (1), 44–49. (In Russian).

[3] Carslaw, H. S., & Jaeger, J. C. (1964). *Teploprovodnost' tverdykh tel [Conduction of heat in solids]*. (In Russian). Nauka. https://onesearch.library.rice.edu/discovery/fulldisplay/alma991009995969705251/01RICE_INST:RICE

[4] Barenblatt, G. I., Entov, V. M., & Ryzhik, V. M. (1972). *Teoriya nestatsionarnoy filtratsii zhidkosti i gaza [Theory of non-stationary filtration of liquid and gas]*. (In Russian). Nedra. https://books.google.com/books/about/Theory_ of_ Fluid_Flows_Through_Natural_Ro.html?id=THr6kQAACAAJ

[5] Boyarchuk, V. T., & Dontsov, K. M. (1971). О кривой восстановления давления в скважине трещинно-пористого коллектора (On the pressure buildup curve in a well of a fractured-porous reservoir). *Journal of Applied Mechanics and Technical Physics, 12*(5), 748–756. (In Russian).

[6] Lavrentiev, M. A., & Shabat, B. V. (1973). *Методы теории функций комплексного переменного (Methods of the theory of functions of a complex variable)*. Nauka. (In Russian). https://www.abebooks.com/Lavrentev-M.A-Shabat-B.V-Methods-theory/32416544408/bd

[7] Jabbarov, I. I., & Safarli, I. I. (2011). Unsteady movement of liquid in a semi-infinite formation in the presence of a feed contour. *Proceedings of the XVII international conference on computational mechanics and modern applied software systems (VMSPPS'2011) (May 25–31, 2011, Alushta)*. MAI-PRINT.

[8] Jabbarov, I. I., Aliyeva, Kh. G., Safarli, I. I., & Gadiyeva, S. S. (2012). Unsteady movement of liquid in a semi-infinite formation in the presence of a fault line. *Proceedings of the IX international conference on nonequilibrium processes in nozzles and jets (NPNJ'2012)*. Moscow: MAI Publishing House. (In Russian).

[9] Jabbarov, I. I., Aliyeva, Kh. G., Safarli, I. I., & Hasanova, Kh. S. (2015). Application of mathematical methods to the solution of some Hydrodynamic problems. *Itogi Nauki: Selected works of the international symposium on fundamental and applied problems of science (issue 19)*. Russian Academy of Sciences.

[10] Safarli, I.I. (2019). *Application of the Laplace transform to the solution of some hydrodynamic problems*. LAMBERT Academic Publishing, 978-613-9-58305-8.